Insurance Inside and Outside the Firm

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Abstract

This paper presents a new equilibrium model with optimal wage contracts, assets, and search frictions. In this model, firms take into account workers' access to financial markets when they design how wages change with tenure and after productivity shocks. The model is consistent with empirical evidence showing that wealthy workers experience higher average wage growth and are more exposed to firm-level productivity shocks than poor workers. The insurance that workers receive outside the firm significantly crowds out the insurance that they receive inside the firm. Specifically, firms provide relatively less insurance to wealthy workers because these workers can self-insure better. Wealthy workers are also matched with more productive firms and receive higher average wages precisely because firms provide less insurance to them. The model has novel implications for public policies that improve the ability of workers to self insure, such as relaxing borrowing constraints.

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1 Introduction

What determines the growth and volatility of worker earnings in the labor market? How can workers insure fluctuations in their earnings to smooth their consumption? The existing literature has tackled these issues from two distinct perspectives. The first is to consider that firms influence the earnings of workers through the design of optimal wage contracts (Baily, 1974, Azariadis, 1975, Thomas and Worrall, 1988, Burdett and Coles, 2003, Guiso, Pistaferri and Schivardi, 2005, Shi, 2009, Menzio and Shi, 2010, Balke and Lamadon, 2022). From this perspective, workers receive insurance inside the firm when they receive wages that are stable over time and across states of the world. The second perspective is to consider that workers use their own assets to smooth consumption over time and in response to income shocks (Bewley, 1977, İmrohoroğlu, 1989, Huggett, 1993, Aiyagari, 1994, Gourinchas and Parker, 2002, Kaplan and Violante, 2014). From this perspective, workers receive insurance outside the firm by trading risk-free bonds subject to borrowing constraints.

Despite decades of research on these issues, there has been no attempt to bring these two perspectives together. The existing models on optimal wage contracts make the stark assumption that workers have no access to financial markets. As a result, firms are the only source of insurance so they have a strong desire to smooth the earnings of workers. These models thus potentially overstate the role of firms as insurance provider. Besides, they cannot account for empirical evidence that wage growth and volatility depend on the wealth of workers (Guiso, Pistaferri and Schivardi, 2012, Fagereng, Guiso and Pistaferri, 2018, Halvorsen, Ozkan and Salgado, 2022). By contrast, the models used in the literature on consumption smoothing make the stark assumption that wage contracts are exogenous. As a result, these models are subject to the Lucas critique (Lucas, 1976) in that the growth and volatility of wages do not respond to changes in policy. Thus, we know very little about the interaction between the insurance that workers receive *inside the firm*, through wage contracts, and *outside the firm*, through financial markets.

This paper builds a new equilibrium model with optimal wage contracts, assets, and search frictions, which brings together the dynamic wage contracting model of Menzio and Shi (2010) with the canonical precautionary savings models in the tradition of Bewley (1977). In this model, firms take into account workers' access to financial markets when they design how wages change with tenure and in response to productivity shocks. I first characterize the optimal contract and then estimate the model using administrative data from France. The model is consistent with empirical evidence showing that wealthy workers experience higher average wage growth and are more exposed to firm-level productivity shocks than poor workers. I find that the insurance that workers receive outside the firm, by trading risk-free bonds, significantly crowds out the insurance that workers receive inside the firm, through wage contracts. Specifically, workers with more assets experience higher wage growth and are more exposed to firm productivity shocks because they can smooth their consumption better. However, despite having more volatile earnings these wealthy workers enjoy a consumption that is more stable over time and less volatile relative to workers with little assets. Thus, wealthy workers receive less insurance inside the firm but more insurance outside the firm and more insurance overall. Besides, wealthy workers are matched with more productive firms and receive higher average wages precisely because firms can optimize worker retention better when they do not provide as much insurance to workers. The model has novel implications for public policies that improve workers' ability to self insure, such as relaxing borrowing constraints. Specifically, enabling workers to borrow reduces the growth rate and volatility of their consumption despite increasing that of their income, and it increases allocative efficiency in the sense that workers are matched with more productive firms and receive higher average wages. This policy is especially effective for relatively poor workers, so it does not only improve efficiency but also reduces income inequality.

The model features risk-averse workers, risk-neutral firms and dynamic wage contracts with directed search. Workers move between employment and unemployment, and can switch jobs. They can trade risk-free bonds subject to a borrowing constraint. Firms select their technology before matching with workers and face firm-level productivity shocks. Firms and workers also face exogenous unemployment shocks. Firms post wage contracts, which specify how wages change over time and in response to shocks. Contracts are subject to two sets of frictions. First, the search decision of workers is private information, which leads to moral hazard. Second, workers and firms cannot commit to transfers after employer-to-employer (EE) separations and to transfers after employment-to-unemployment (EU) separations. The assumption that firms cannot make transfers to workers after EU separations is meant to capture that not all risk is insurable by firms, so that assets are used for precautionary savings. In the baseline model, the savings decision of workers is public information so firms and workers can contract on it but I also consider the model in which savings are private information and show that it makes quantitatively no difference.

I first show that allowing workers to trade assets influences optimal contracts, even when assets are public information, precisely because workers and firms cannot commit to transfers post EE or EU separations. Specifically, if firms and workers could commit to transfers post EE or EU separations, then the optimal contract can be implemented with zero asset holdings for workers. This means that the assumption that workers are hand-to-mouth is not restrictive in this case. Besides, I show that the optimal contract can use assets as a substitute for either type of transfers. For example, instead of implementing a transfer from workers to firms after EE separations, the contract can be implemented by reducing the worker's assets holdings and backloading wages. This ensures that the continuation value of workers falls after EE separations. Conversely, instead of implementing a transfer from firms to workers after EU separations, the contract can be implemented by increasing the worker's assets and frontloading wages. This ensures that the worker is insured against unemployment risk. In the quantitative model, neither type of transfers can be implemented so assets are used to substitute for both, leading to a trade-off where assets are used to enhance worker retention and to insure workers against unemployment risk[1].

This trade-off between worker retention and insurance can be characterized using the optimality conditions of the optimal contract. The first is a standard consumption growth condition, which also arises in optimal wage contracts with hand-to-mouth workers (e.g. Balke and Lamadon, 2022). The second is a new pseudo Euler equation, which highlights the different roles that assets play in optimal wage contracts. This equation shows that the optimal contract will use assets to smooth the worker consumption over time and across states, for a given path of wages. For instance, the contract will deplete the worker assets when wages are backloaded to smooth consumption over time. By contrast, the contract will increase the worker assets when workers face unemployment risk, to increase the precautionary savings of workers. The pseudo Euler equation also shows that firms manipulate assets to influence the search decision of workers because of wealth effects, as in Acemoglu and Shimer (1999), and because assets influence the set of contracts available to workers at the next jobs. However, these effects are quantitative very small and thus do not influence optimal contracts much.

Consider how the presence of assets changes the optimal degree of wage backloading in optimal wage contracts, and thus the average wage growth. In designing contracts, firms face a trade-off between retaining and attracting workers. Remember that firms cannot control the worker search decision because it is private information. As a result, the optimal retention strategy for firms is to backload wages to make workers search

[1] These results clarify why assets play a role in my model whereas they do not in the literature on optimal unemployment insurance (e.g. Hopenhayn and Nicolini, 1997). In this literature, it is standard to assume that the government (the principal) can tax the worker (the agent) when she finds a job. This assumption is equivalent to assuming that transfers post EE separations can be implemented. There is no exogenous separation between the government and the worker so transfers for EU separations play no role. Hence the focus of this literature on hidden saving.

for jobs with higher wages and lower job finding rates as in Burdett and Coles (2003) and Shi (2009). Firms know that in response to backloaded wages, workers will smooth their consumption over time by consuming their existing assets. Therefore, relative to a model without assets firms can potentially backload wages significantly more. However, firms also know that workers face borrowing constraints so they might not be able to smooth consumption over time. Besides, they know that workers would like to save to self-insure against unemployment risk. In particular, backloading wages too much and making workers consume their savings at the beginning of a match makes contracts very unattractive to risk-averse workers who anticipate that their consumption will fall significantly if an unemployment shock occurs. Indeed, if one firm adopted a strategy of extreme wage backloading, it would have to offer much higher average wages to make its offer attractive relative to an offer that offers more stable wages and, hence, more stable consumption. The optimal contract thus balances the desire of firms to retain workers through backloaded wages, and the desire of workers to smooth consumption and self-insure against unemployment risk through frontloaded wages. Relative to a model with hand-to-mouth workers, this model therefore generates wage paths that can be more or less backloaded, and even frontloaded, and that depend on the worker's existing assets and ability to borrow.

Introducing assets into the model also influences how firms choose to pass productivity shocks through to workers. This is important because firm-level productivity shocks constitute an important source of income volatility for workers (Guiso et al., 2005). First, the pass-through to wages is larger when workers have access to financial markets than when they are hand-to-mouth. Second, and perhaps more surprisingly, firms also take advantage of workers' access to financial markets to influence the shape of the pass-through, and not just its size. To understand how, consider how firms respond to a positive persistent shock to productivity. As in model with hand-to-mouth workers, firms respond by increasing the wage of workers to reduce the quit rate. In contrast however, firms now respond with promises of higher wages in the future, thus backloading wage increases. In fact, if shocks are sufficiently persistent, the optimal response to a positive shock is to cut wages on impact and later increase wages significantly. Meanwhile, consumption increases smoothly over time as workers initially deplete their assets in anticipation of higher future income. Why is it optimal for firms to cut wages and deplete the worker assets on impact after a positive shock? When productivity increases, the profits of firms rise so the worker retention motive described before becomes stronger relative to the insurance motive so firms choose to backload wages more. They backload wages so much that wages can actually fall on impact.

I estimate the model using French administrative data to quantify how much insurance workers across the wealth distribution receive inside and outside the firm. The model is consistent with empirical evidence that wages grow on average with tenure and with evidence that both the pass-through of productivity shocks to wages and the marginal propensity to consume are positive but less than one. The model is also consistent with existing empirical evidence showing that the wage of asset-rich workers grows more on average and responds more to firm-level productivity shocks relative to asset-poor workers. Relative to a model with hand-to-mouth workers, I find that workers receive less insurance from firms in that their wages are more backloaded and the pass-through of productivity shocks to wages is higher. In this sense, the insurance that workers receive outside the firm crowds out the insurance that they receive inside the firm. Workers however receive more insurance overall relative to hand-to-mouth workers because their consumption is more stable over time and responds less to productivity shocks.

The model implies that wage growth is heterogeneous over the wealth distribution. Specifically, workers who start their career with relatively more assets receive wages that are more backloaded because they can smooth their consumption themselves. As a result, they experience higher wage growth but lower consumption growth than workers who start their career with relatively less assets. Wealthy workers also match with more productive firms and receive higher average wages because firms invest in better technology when they can backload wages more and worker mobility rates are low. Quantitatively, I find that the wages of workers at the top of the wealth distribution grows by 0.5% more on average per year and is 1% higher relative to workers at the bottom of the wealth distribution. This mechanism is consistent with evidence that workers with better access to financial markets receive wages that are more backlaoded (Guiso et al., 2012) and that workers with wealthy parents experience higher wages growth during their career than workers with poor parents (Halvorsen et al., 2022).

The model also implies that the pass-through of productivity shocks is heterogeneous over the wealth distribution. Specifically, the pass-through to wages is about twice larger for workers at the top of the wealth distribution relative to workers at the bottom of the distribution. The pass-through to consumption however is 3 times lower for workers at the top relative to workers at the bottom because these workers have a much lower MPC. This shows that wealthy workers receive less insurance from firms but more insurance overall against shocks. The reason why firms do not provide more insurance to poor workers is that these workers are at the bottom of the job ladder. As a result, their EE rate is high so employers are willing to let their consumption fluctuates more to optimize worker retention relative to workers at the top of the job ladder whose EE rate is much less sensitive to their consumption. This mechanism is consistent with empirical evidence from Fagereng et al. (2018) who show that the pass-through of firm-level productivity shocks to wages is increasing in wealth.

Taken together, these results have new implications for public policies because they show that the wealth of workers influences how much insurance workers receive against shocks and where this insurance is coming from. In particular, policies that relax the borrowing constraint of workers improve their ability to self-insure, just like wealthy workers. As a result, workers receive less insurance from firms in that their wages become more backloaded and more exposed to firm productivity shocks but they receive more insurance overall in that their consumption becomes less backloaded and respond less to shocks. Quantitatively, I find that letting workers borrow up to 2 quarters of income increases average wage growth 4 times but reduces average consumption growth 3 times, and it increases the pass-through to wages by 40% but reduces the pass-through to consumption by 30% for workers with less than 10 years of experience who are most affected by the policy. This policy also improves allocative efficiency as workers match with firms that are 1.5% more productive on average and receive average wages that are 0.6% higher.

Throughout, I have assumed that firms can observe the initial assets of newly matched workers and use this information to design wage contracts. The standard justification for this assumption is that firms can infer the worker's assets based on observable characteristics, such as occupation. I show that even if firms cannot do this, they can still get this information because in equilibrium workers with different initial assets select contracts with different characteristics. In this sense, the equilibrium is robust to this assumption. I establish this result using the methodology of Guerrieri, Shimer and Wright (2010) to solve models with directed search and adverse selection.

In this paper I focus on non-contingent assets, as opposed to more general securities, because in the data most assets held by households, such as cash, are non-contingent. This assumption has two important implications relative to the model where workers have access to complete financial markets studied by Stevens (2004). First, with complete markets the optimal contract no longer suffers from moral hazard. In particular, the worker buys the job from the firm by paying an upfront fee, and then receives a wage equal to the value of output. By contrast, with non-contingent assets, such a contract is not optimal because firms want to insure workers against the risk of not finding another job. As a result, the optimal contract is still subject to moral hazard[2]. Second, the

[2]This is similar to what Shimer and Werning (2008) call the need to "insure against uncertain spell

model with complete markets makes predictions that are sharply at odds with the data. In particular, in my model the tenure profile of wages is consistent with the data precisely because workers face uninsurable unemployment risk, and there is limited pass-through of productivity shocks to wages, as in the data, only with incomplete markets.

This paper contributes to a recent literature bringing together labor market transitions and asset accumulation. An early example is Krusell, Mukoyama and Şahin (2010) who study precautionary savings in a DMP model where wages are set by Nash bargaining. Several articles have since introduced assets in search models with EE separations (Lise, 2013, Chaumont and Shi, 2022, Alves, 2022, Kaas, Lalé and Nawid, 2023, Caratelli, 2024) but in these models wage contracts are subject to ad hoc restrictions. Specifically, wages are assumed to be constant during matches, or assumed to change only when workers receive an outside offer. Instead, my paper is the first to study the determinants of worker mobility and assets in a model with optimal wage contracts. The main advantage of my approach is that it allows to study the sources of insurance that workers receive against labor market risk. Besides, I also show that fixed-wage contracts are inefficient in my environment because they abstract from the firm's desire to retain workers and the worker's demand for precautionary savings. Quantitatively, I find the gains from optimal contracts relative to fixed-wage contracts for firms to be quite large.

The paper starts in section 2 by presenting a new model with wage contracts and assets, which I characterize in section 3. Section 4 brings the model to data and quantifies determinants of income inequality and the amount of insurance that workers receive inside and outside the firm. Finally, section 5 revisits the assumption that firms can observe the initial assets of newly hired workers. Proofs are in the appendix.

2 A model with wage contracts and assets

I first present a new model with search frictions, dynamic wage contracts and assets. The model combines the optimal contract with search frictions of Menzio and Shi (2010) and the precautionary savings model of Bewley (1977).

2.1 Environment

Time is discrete and runs forever.

duration" in the unemployment insurance literature.

Agents A continuum of ex-ante homogeneous workers can be employed or unemployed. Workers receive wage w when employed, and home production b when unemployed. They have period utility $u(c)$ over consumption and discount the future at rate β.

Firm are owned by foreign diversified investors, so they are effectively risk-neutral with discount rate r. An active firm is one that is matched with a single worker. The output from that match x_t follows the mean reverting process

$$x_t = (1-\rho)x_0 + \rho x_{t-1} + \sigma \nu_t$$

where ν_t are i.i.d. innovations with standard normal distribution, and ρ parametrizes the persistence of productivity. The mean productivity x_0 is selected by firms before they match with workers, and should be interpreted as the result of firm investment in worker training and in production technology. This component stays constant over time and lasts for the length of the match.

Financial markets This is a small open economy with foreign interest rate r. Workers can save using risk-free bonds a_{t+1} subject to a borrowing constraint, so that

$$a_{t+1} \geq 0$$

Timing Each period, the sequence of events is as follows

a) Productivity shocks ν_t and exogenous separations into unemployment occur

b) Employed workers and workers who were unemployed at the start of the period search for jobs; firms post vacancies; new matches are formed

c) Firms produce and pay current wages; workers make saving decision and consume

Directed search with assets There is a continuum of labor markets indexed by the promised value to a worker denoted v and the assets of workers a. Indexing labor markets by the worker promised value v is critical so that workers know where to search for a job. Indexing labor markets by the worker assets a is critical so that firms know how much profit they will generate from a match. Every period, workers choose in which labor market to search and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets.

I assume throughout that firms commit to delivering value v in markets indexed by v. I also assume in the baseline model that workers with assets a can only search in markets indexed by a, and not elsewhere. In section 5, I will consider the possibility that workers with assets a search in markets indexed by $\tilde{a}$ so that firms might not know how much assets the workers they match with actually have.

Denote $\phi_u(v,a)$ and $\phi_e(v,a)$ the mass of unemployed and employed workers searching for a job and denote $\phi_f(v,a)$ the mass of vacancies posted by firms. Let κ denote the search intensity of employed workers relative to unemployed workers. In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)$ turns workers searching for a job and vacancies into matches. Define the job finding rate $\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that an unemployed worker finds a job, and the vacancy filling rate $\tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that a vacancy finds a worker. These probabilities are defined in the usual way as

$$\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_u + \kappa\phi_e}, \quad \tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_f}$$

Since these matching probabilities will depend on v and a in equilibrium, we can write them in short-hand notation as

$$\lambda_w(v,a) \equiv \tilde{\lambda}_w(\phi_u(v,a) + \kappa\phi_e(v,a), \phi_f(v,a)), \quad \lambda_f(v,a) \equiv \tilde{\lambda}_f(\phi_u(v,a) + \kappa\phi_e(v,a), \phi_f(v,a))$$

Unemployed workers Unemployed workers face a consumption-savings decision problem similar to Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023). They receive an endowment b, choose how much to save and consume and in which labor market v to search. Given the job finding probability, $\lambda_w(v,a)$, the value of unemployed workers satisfies

$$\begin{aligned} U(a_t) = \max_{c_t, a_{t+1}, v_{t+1}} \quad & u(c_t) + \beta\left[\lambda_w(v_{t+1}, a_{t+1})v_{t+1} + (1 - \lambda_w(v_{t+1}, a_{t+1}))U(a_{t+1})\right] \\ \text{s.t.} \quad & c_t \leq (1+r)a_t + b - a_{t+1} \\ & a_{t+1} \geq 0 \end{aligned}$$

Appendix A.4 shows that the choice of savings a_{t+1} follows a standard Euler equation where workers smooth their consumption by depleting their savings over time. The choice of search v_{t+1} follows a standard trade-off: searching in a high-v labor market brings a higher value v conditional on a match, but it will turn out that these matches occur with lower probability because $\lambda_w(v,a)$ will decrease with the value v in equilibrium[3].

[3] In models without assets and where x_0 is homogeneous across firms, the search policy exists and is unique (see Menzio and Shi, 2010) because the job finding rate $\lambda_w(v)$ is concave in equilibrium. In my

Employed workers Employed workers find a new job in market v when their current asset is $a_t$with probability $\kappa\lambda_w(v, a_t)$. Existing matches break up and workers separate into unemployment with exogenous probability δ[4].

Contracts Optimal wage contracts specify wages w_t and transfers after separations into employment $(1+r)\tau_t^{ee}$ or unemployment $(1+r)\tau_t^{eu}$ for each history of shocks and conditional on the worker's initial asset position a_0 and on delivering initial value v to workers. Productivity is public information and I assume that it is unfeasible for firms to make counteroffers to their workers when they receive outside job offers[5]. In the baseline model, I assume that the savings decision of workers a_{t+1} is public information so it is contractible but I also solve a version of the model in which the savings decision is private information and find that the results are nearly identical, even though this version is much harder to solve and characterize.

The contract is subject to two sets of contracting frictions, which both play a critical role in the analysis. First, the worker search decision is private information, which leads to moral hazard. Second, workers and firms cannot commit to transfers post separations into employment τ_t^{ee} or unemployment τ_t^{eu}. This assumption implies that these transfers must be equal to 0 after any history in the optimal contract.

I show in section 3.1 that letting workers trade risk-free bonds influences the optimal allocation precisely because these transfers post EE and EU separations cannot be implemented. The assumption that transfers post EE separations cannot be implemented is standard in the literature on optimal contract and is usually justified on the ground that bonded labor is prohibited by law (e.g. Stevens, 2004). The assumption that transfers post EU separations cannot be implemented is more controversial because firms do make severance payments in practice. However, even formal commitment about severance pay can be subject to interpretation[6]. Besides, firms might not able to implement severance payments to workers if the separation occurs because of firm bankruptcy. Beyond this

model however, the search and savings policies might not be unique because the equilibrium job finding rate $\lambda_w(v, a)$ is not always concave as firms select their average productivity x_0 and because there might be complementarity between the search and savings decision of workers. I solve the model using the optimality conditions of the worker and verify ex-post that the solution corresponds to the unique solution numerically.

[4]To keep the notation simple, I do not allow for quits into unemployment. However, I verify ex-post that this restriction does not bind in the quantitative model. I find that because the endowment value b is well below firm productivity, almost no worker would be better off quitting in model-simulated data.

[5]This assumption can be formally justified as follows: counteroffers are private information to workers, and expire before workers can return to their current employers to negotiate higher wages. These assumptions ensure that it is optimal for current employers not to respond to outside offers.

[6]For example, severance pay only applies if termination occurs for reasons outside of worker's control.

specific assumption, what is critical for the analysis is that workers face some risk that firms cannot insure directly and therefore require insurance outside firms in the form of precautionary savings[7].

2.2 Optimal contracts

Following previous work on dynamic contracts, I write the contract recursively in terms of promised values and continuation values instead of histories of shocks[8]. Denote V_t the promised value of an employed worker at the start of the period. The state of a match at the beginning of the period is the worker promised value V_t, the asset of the worker a_t, the firm average productivity x_0 and current productivity x_t. Denote by $s_t \equiv V_t, a_t, x_0, x_t$ the vector of current state variables.

The components of the contract at time t are the wage paid today, the transfers post EE and EU separations, the savings decision of workers and a set of continuation values for each state tomorrow. Formally, these components are represented by the functions

$$w_t\left(s_t\right), \quad \tau_t^{ee}\left(s_t\right), \quad \tau_t^{eu}\left(s_t\right), \quad a_{t+1}\left(s_t\right), \quad V_{t+1}(s_t, x_{t+1})$$

A *contract* is a collection of these functions for all t. In the recursive formulation below, we write the components of the contract as $w_t, \tau_t^{eu}, \tau_t^{ee}, a_{t+1}, V_{t+1}(x_{t+1})$ without explicitly mentioning state variables. It will also be convenient to define the continuation value of workers at the current job as

$$W_t \equiv u(c_t) + \beta \mathbb{E}_{x_{t+1}}\left[V_{t+1}(x_{t+1}) | x_0, x_t\right] \tag{1}$$

where $c_t + a_{t+1} = (1+r)a_t + w_t$.

[7]There are other ways to generate such need for insurance outside the firm. For instance, if workers receive income shocks that are private information to workers (e.g. health expense shocks, spousal income shocks), then the amount of insurance that firms can provide is limited (as in Cole and Kocherlakota, 2001). In addition, firms could be subject to walk-away constraints specifying that their continuation value must remain positive, which implies that the wage of workers will fall after large negative shocks (as in Thomas and Worrall, 1988). In this case, workers would want to accumulate precautionary savings against this risk. Finally, workers might also need to accumulate assets outside of firms for the purchase of durable goods such as housing. In this paper I focus on a simple and yet large reason why workers need to accumulate assets outside of firms: unemployment risk.

[8]I abstract from randomized contracts to keep notation simple.

Worker value Given the contract, the worker chooses a search strategy to maximize the present value of utility. The value of a worker satisfies

$$V_t = \delta U(a_t + \tau_t^{eu}) + (1-\delta) \max_{v_t} \left[\kappa \lambda_w(v_t, a_t + \tau_t^{ee}) v_t + (1 - \kappa \lambda_w(v_t, a_t + \tau_t^{ee}))\, W_t\right] \quad (2)$$

In equation (2), the first term is the continuation value of a worker who becomes unemployed at time t with assets a_t and receive transfer from firms τ_t^{eu}. The second term depends on the probability that a worker finds another job $\kappa\lambda_w(v_t, a_t + \tau_t^{ee})$ and on the value that the worker receives if an EE separation occurs v_t. A worker with asset a_t who finds a new job will start that job with asset $a_t + \tau_t^{ee}$. A worker who does not find a new job receives the continuation value W_t.

Equation (2) shows how the assumptions of hidden search and limited commitment about transfers interact. Notice that moral hazard arises because the worker's search decision v_t only depends on the surplus that workers get from EE separations $v_t - W_t$, and not on the firm value. Thus, relative to the search policy that the worker chooses, a firm with a positive value would prefer the worker to search instead in markets with a higher value v_t and a lower job finding rate $\lambda_w(v_t, a_t + \tau_t^{ee})$ because it wants to retain the worker. Since the worker search decision v_t is private information, the firm cannot control it directly and instead influences the worker's decision indirectly by manipulating the continuation value at the current job W_t and the job finding rate $\lambda_w(v_t, a_t + \tau_t^{ee})$. How can the firm manipulate the job finding rate? If firms and workers could commit to transfers, the firm could enforce a transfer τ_t^{ee} from the worker after EE separations. This would reduce the worker's asset at the next job, thus lowering the job finding rate in equilibrium. When these transfers cannot be implemented, the firm can instead manipulate the worker's assets a_t. However, assets also influence the continuation value of workers who become unemployed $U(a_t + \tau_t^{eu})$ so firms optimally manipulate the workers' assets to influence their EE mobility only to some extent. This dual role for assets generates a trade-off that I describe in details in section 3.1.

Optimal contracts Denote the optimal search policy $v(W_t, a_t + \tau_t^{ee})$[9], the implied EE probability as

$$p_t \equiv p(W_t, a_t + \tau_t^{ee}) \equiv \kappa \lambda_w(v(W_t, a_t + \tau_t^{ee}), a_t + \tau_t^{ee})$$

[9]As for unemployed workers, the search decision might not be unique in this environment. I check it ex-post numerically when I solve the model.

and the worker expected surplus from EE separations as

$$S_t \equiv S(W_t, a_t + \tau_t^{ee}) \equiv \kappa \lambda_w(v(W_t, a_t + \tau_t^{ee}), a_t + \tau_t^{ee}) \left(v(W_t, a_t + \tau_t^{ee}) - W_t\right)$$

Finally, denote $\Pi(s_t)$ the present value of profits for a firm matched with a worker who has promised value V_t and assets a_t when average productivity is x_0 and currently x_t. Taking as given the value of unemployment $U(a)$, the equilibirum job finding rate $\lambda_w(v, a)$, and the search policy of workers $v(W, a)$, the optimal contract solves

$$\Pi(s_t) = \max_{w_t, \tau_t^{eu}, \tau_t^{ee} c_t, a_{t+1}, V(x_{t+1})} (1-\delta)(1 - p(W_t, a_t + \tau_t^{ee})) \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}[\Pi(s_{t+1})|x_0, x_t]}{1+r} \right) \\ -\delta(1+r)\tau_t^{eu} - (1-\delta)p(W_t, a_t + \tau_t^{ee})(1+r)\tau_t^{ee} \tag{3}$$

subject to

$$\begin{aligned} &(\text{PK}): && V_t \leq \delta U(a_t + \tau_t^{eu}) + (1-\delta)\left[W_t + S(W_t, a_t + \tau_t^{ee})\right] \\ &(\text{Budget}): && c_t + a_{t+1} = (1+r)a_t + w_t \\ &(\text{BC}): && a_{t+1} \geq 0 \\ &(\text{LC}): && \tau_t^{eu}, \tau_t^{ee} = 0 \end{aligned}$$

where $W_t = u(c_t) + \beta \mathbb{E}_{x_{t+1}}[V_{t+1}(x_{t+1})|x_0, x_t]$ and $s_{t+1} \equiv V(x_{t+1}), a_{t+1}, x_0, x_{t+1}$.

The contract maximizes the present value of profits, where $(1-\delta)(1 - p(W_t, a_t + \tau_t^{ee}))$ is the probability that the worker remains within the current match this period. The first constraint (PK) is the promise keeping constraint, stating that the value the worker gets from the contract either at the current job, through unemployment or at future jobs must deliver at least the promised value V_t. The second constraint (Budget) is the budget constraint of the worker, and the third constraint (BC) is the borrowing constraint. The last constraint (LC) is the worker and firm limited commitment constraint. We wrote the optimal contract taking as given the optimal search policy of workers $v(W, a)$, so the incentive compatibility constraint for search is implicit in the definition of $p(W, a)$ and $S(W, a)$.

Value of new matches In the first period of employment, firms also solve (3) except that there is no EU separations ($\delta = 0$), no EE separations ($p_t = S_t = 0$) and no productivity shock ($\nu_t = 0$). Denote the firm value in the first period by $\Pi_0(V_t, a_t, x_0)$.

2.3 Equilibrium

Free entry Firms post vacancies in each labor markets subject to a free entry condition. Firms choose in which market (v, a) to post vacancies and which technology x_0 to adopt. The unit cost of posting a vacancy with technology x_0 is $k(x_0)$ with $k, k', k'' > 0$. The free entry condition is

$$\max_{x_0} \; -k(x_0) + \lambda_f(v, a)\Pi_0(v, a, x_0) \leq 0 \tag{4}$$

with equality for each active market (v, a).

Making the choice of x_0 endogenous is a standard way to generate fixed heterogeneity in productivity across firms (Acemoglu and Shimer, 2000), which is widely accepted as one of the main drivers of worker mobility. In equilibrium, firms that post vacancies in markets with higher values v or lower assets a will select higher productivity. A convenient implication of this formulation is that it increases the upper bound on the worker value v for active markets. This means that even workers with relatively high wages can search in markets with higher values, where firms are very productive and where the job finding rate is very low. As a result, even these workers can switch jobs, which is critical to generate some of the quantitative results from section 4.3. By contrast, when x_0 is fixed across firms, workers with relatively high wages arrive quickly at the top of the job ladder where their EE separation rate is 0[10]. The assumption that firm initial productivity x_0 is endogenous also turns out to interact with the optimal contract and the worker's initial asset holding, as I explain in section 4.2.

Definition of an equilibrium An equilibrium is a set of value functions, policies and matching rates for each labor market (v, a) such that i) the unemployed worker policies maximizes the unemployment value, ii) the firm and employed worker policies satisfy the optimal contract, iii) the free entry condition is satisfied and iv) the job finding and vacancy filling rates are consistent with the matching function. The laws of motion for the distributions $D^u(a)$ and $D^e(s)$, defined as usual, are satisfied given the policies.

3 The role of assets in wage contracts

I now characterize the optimal wage contract by describing how wages, assets and consumption change with tenure and in response to productivity shocks. I will emphasize

[10] Alternative ways to make the job ladder longer include making the EU separation probability δ endogenous as in Balke and Lamadon (2022) or assuming that workers receive iid preference shocks for jobs that are private information as in Souchier (2023).

how assets alter contracts relative to models with optimal wage contracts and hand-to-mouth workers (as in Burdett and Coles, 2003) and relative to models with hidden search from the unemployment insurance literature where assets play no role (as in Hopenhayn and Nicolini, 1997). I first clarify that assets influence optimal contracts because they substitute for missing transfers post EE and EU separations. Because firms cannot implement those transfers, they decide to use workers' assets to optimize worker retention further but also to insure them against unemployment risk. I then use the optimality conditions of the optimal contract, including a new pseudo Euler equation that I derive, to show how the paths of wages, assets and consumption depend on this new trade-off between worker retention and insurance.

3.1 Assets substitute for missing transfers

This section clarifies that assets influence optimal contracts because they substitute for missing transfers post EE and EU separations. I state the main result in proposition 1 and then explain it using figure 1 for illustration.

Proposition 1. *Consider the following 3 cases regarding transfers τ_t^{ee} and τ_t^{eu}:*

1. Assume that firms and workers can commit to both τ_t^{ee} and τ_t^{eu}, that is the optimal contract (3) does not need to satisfy constraint (LC). Then, the path of consumption and EE probability are identical in optimal contract with assets (3) and in a restricted contract with hand-to-mouth workers, that is with the additional constraint $a_{t+1} = 0$. Denote the optimal paths of consumption and EE probability as c_t^*, p_t^* and denote the paths of transfers that implement the optimal contract with $a_{t+1} = 0$ as $(\tau_t^{ee})^*$ and $(\tau_t^{eu})^*$.

2. Assume instead that firms and workers can commit to τ_t^{eu} but not to τ_t^{ee}, that is constraint (LC) in the optimal contract (3) is replaced by $\tau_t^{ee} = 0$. Assume further that productivity is constant within matches and that workers face no borrowing constraint. Then, the paths of consumption and EE probability are identical to case 1, that is $c_t = c_t^*$ and $p_t = p_t^*$ for all t. Besides, the optimal path for assets satisfies

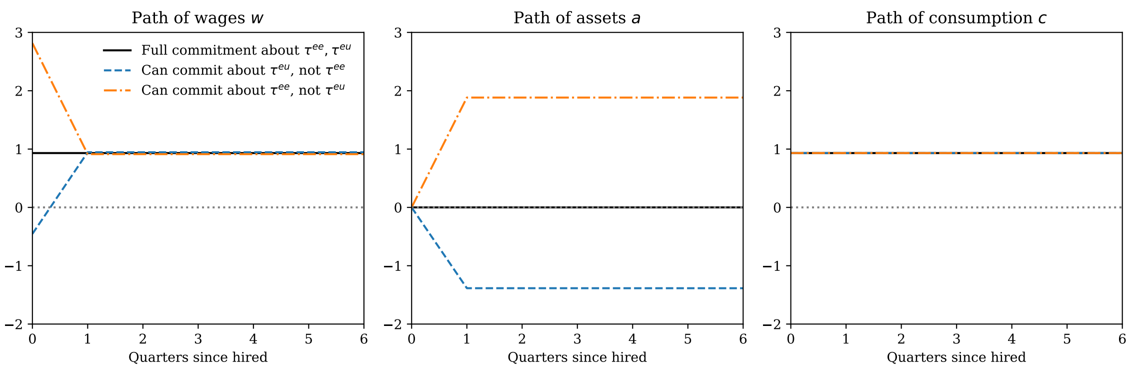
$$a_{t+1} = (\tau_{t+1}^{ee})^* \quad \forall t$$

3. Assume instead that firms and workers can commit to τ_t^{ee} but not to τ_t^{eu}, that is constraint (LC) in the optimal contract (3) is replaced by $\tau_t^{eu} = 0$. Assume further that productivity is constant within matches and that workers face no borrowing constraint. Then, the paths of consumption and EE probability are identical to case 1, that is $c_t = c_t^*$ and $p_t = p_t^*$ for all t. Besides, *the optimal path for assets satisfies*

$$a_{t+1} = (\tau^{eu}_{t+1})^* \quad \forall t$$

Proof. See appendix A.1. □

Figure 1: Assets substitute for missing transfers post EE and EU separations



Note: paths of wages, assets and consumption for an employed worker hired from unemployment at $t = 0$ with zero initial asset. The solid black line shows the model where both transfers τ^{ee} and τ^{eu} can be implemented, and where it is assumed that $a_{t+1} = 0$. The dashed blue line shows the model where τ^{eu} but not τ^{ee}. The dash-dotted orange line shows the model where τ^{ee} but not τ^{eu}. In all models, productivity is assumed to be constant across firms and over time ($x_t = 1$) and workers and firms have the same discount factor $\beta(1 + r) = 1$.

Case 1 from proposition 1 shows that the optimal allocations c_t and p_t are identical whether workers are hand-to-mouth or can trade risk-free bonds, provided that firms and workers can commit to transfers τ^{ee}_t and τ^{eu}_t. In this case, letting workers trade risk-free bonds is therefore irrelevant when these trades are public information. This is a well known result in the optimal unemployment insurance literature: the principal can implement the optimal contract in many different ways, including by saving on behalf of the agent and setting $a_{t+1} = 0$ for the duration of the match (e.g. see Werning, 2002). Critically, this result requires that transfers τ^{ee}_t and τ^{eu}_t can be chosen freely in the optimal contract. This assumption is in fact standard in the literature on optimal unemployment insurance[11], but not in the literature on optimal wage contracts[12]. This is the reason why

[11]Since Hopenhayn and Nicolini (1997), it is common practice in this literature to assume that the government (the principal) can tax workers (the agent) when they find a job. This assumption is equivalent to assuming that workers can commit to transfers when they find a job, that is τ^{ee}_t can be freely chosen. Furthermore, in these models there is no exogenous separation between the government and unemployed workers ($\delta = 0$) so τ^{eu}_t is irrelevant. Interestingly, the seminal article of Shavell and Weiss (1979) did not assume that the government could tax workers but the literature on hidden savings has taken the model of Hopenhayn and Nicolini (1997) as a starting point instead.

[12]The literature on optimal wage contracts (e.g. Burdett and Coles, 2003; Shi, 2009) assumes that workers

letting workers trade risk-free bonds influences optimal wage contracts even when they are public information whereas they do not influence optimal unemployment insurance contracts. The solid black line from figure 1 shows one implementation of the optimal contract, the one satisfying $a_{t+1} = 0$ for all t. In this specific example, productivity is constant across firms and over time, the worker is matched from unemployment with zero initial asset and $\beta(1+r) = 1$. The optimal contract implements a constant path for consumption with a *constant wage*.

Case 2 from proposition 1 shows that the optimal allocations c_t and p_t remain the same as in case 1 even if firms and workers can only commit to transfers post EU separations τ_t^{eu} but not to transfers post EE separations τ_t^{ee}, provided that workers can trade risk-free bonds. In this sense, the optimal contract uses assets as a substitute for transfers τ_t^{ee} to influence worker mobility decisions. The dashed blue line from figure 1 shows how the optimal contract is implemented in this case. Wages are now *backlaoded*, and in fact negative during the first period of employment. The optimal contract makes the worker borrow and consumption is constant over time, as in case 1. Therefore, this figure confirms that the optimal allocation is identical in cases 1 and 2 even though the implementations of the contract differ.

To understand why assets can be used to substitute for transfers post EE separations τ_t^{ee}, it is useful to consider how the optimal contract uses these transfers in the first place. The main contracting friction in the optimal contract is that the search decision of workers is private information. This means that the worker's search decision, and the implied EE separation rate, are chosen to maximize the worker's value, not the joint value from the match. Often, in the context of optimal wage contracts, this means that workers are too likely to switch jobs so firms want to reduce worker mobility. When firms and workers can commit to transfers, the optimal contract thus requires workers to pay a fee back to their previous employer when they switch jobs, that is $\tau_t^{ee} < 0$. This reduces the worker's continuation value at the next job and ensures that the worker's search decision maximizes the joint value. When these transfers cannot be implemented, the optimal contract instead backload wages to reduce the worker's assets as in figure 1. This too reduces the worker's continuation value at the next job and ensures that the worker's search decision maximizes the joint value[13].

cannot commit to transfers post EE separations, τ_t^{ee}, not only because it is realistic but also because it is critical to generate wages that are backloaded, which is a key prediction of these models. If workers could commit to τ_t^{ee}, then wages would be either constant or frontloaded.

[13]In this specific example, introducing assets solves the model hazard because productivity is constant across firms. With heterogeneity in productivity, wages and consumption would not be constant over time even with transfers. This is because with constant wage the consumption of a worker matched with a low-productivity firm would jump up when she finds a better job. It is thus optimal for firms to insure

Case 3 from proposition 1 considers the opposite assumptions about transfers. It shows that the optimal allocations c_t and p_t remain the same as in case 1 even if firms and workers can only commit to transfers post EE separations τ_t^{ee} but not to transfers post EU separations τ_t^{eu}. Here, the optimal contract uses assets as a substitute for transfers τ_t^{eu} to insure workers against unemployment risk. The dash-dotted orange line from figure 1 shows how the optimal contract is implemented in this case. Wages are now *frontloaded*. The optimal contract makes the worker save and consumption is constant over time, as in case 1. As before, the allocations are the same as in case 1 but the implementations differ.

To understand why assets can be used to substitute for transfers post EU separations τ_t^{eu}, consider first how firms provide insurance to workers when these transfers are available. In this case, firms implement a transfer in the event of an exogenous separation into unemployment, effectively giving workers severance payment to insure them against unemployment risk. When these transfers cannot be implemented firms instead help workers self-insure against unemployment by making sure that workers have enough assets to smooth consumption themselves if they become unemployed. This is achieved by frontloading wages in the first period, which can be interpreted as paying workers a hiring bonus. In fact, the optimality conditions of the optimal contract show that firms achieve perfect insurance this way because $(1+r)\beta u'(c_{t+1}^u) = u'(c_t)$, meaning that the marginal utility of consumption remains constant after unemployment shocks.

Cases 2 and 3 also make the additional assumptions that workers should be able to borrow and that productivity is constant within matches. The reason for letting workers borrow is that transfers can be negative so replicating them implies to reduce the worker's assets. When the worker's initial asset is low, this may require workers to borrow. The reason for productivity to remain constant is that transfers are chosen after shocks are realized whereas assets a_{t+1} are non-contingent and chosen in the previous period[14].

Taken together these results show that assets are used to substitute for missing transfers post EE and EU separations. When transfers post EE separations τ_t^{ee} cannot be implemented, firms use worker's assets to optimize worker retention. When transfers post EU separations τ_t^{eu} cannot be implemented, firms use worker's assets to help them self-insure against unemployment risk. In the model from section 2, neither τ_t^{ee} nor τ_t^{eu} can

the worker against this risk by increasing consumption at the current job, and reducing assets to reduce consumption at the future job. However, because the worker search decision is still private information, this brings back moral hazard. The optimal contract thus reduces the consumption and assets of workers over time, in a similar way as the government reduces unemployment benefits and increases taxes with unemployment duration in Hopenhayn and Nicolini (1997).

[14] These conditions are really binding only for case 2, as figure 1 illustrates, because in case 3 the optimal contract usually implies a transfer from firms to workers and because τ_t^{ue} are independent of current productivity.

be implemented so firms face a trade-off when they choose how to use worker's assets in the optimal contract. The next section characterizes this trade-off using the optimal conditions of the contract (3).

3.2 Implications for tenure profiles

The previous section showed that letting workers trade risk free bonds leads to a trade-off between optimizing worker retention and helping worker self-insure against unemployment risk. This section uses the optimality conditions of the contract (3), including a new pseudo Euler equation, to characterize this trade-off and emphasize new implications relative to wage contracts with hand-to-mouth workers. In this section, we focus on tenure profiles that describe how wages, consumption and assets change with tenure at the firm even when productivity remains constant. In the next section, we will discuss how firms pass productivity shocks through to workers.

The consumption growth condition Our starting point is the following consumption growth condition

$$\frac{1}{u'(c_t)} - \frac{\beta(1+r)}{u'(c_{t-1})} = -\frac{p_W(W_t, a_t)}{1-p(W_t, a_t)} \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}} \left[\Pi(s_{t+1}) | x_0, x_t \right]}{1+r} \right) \tag{5}$$

which is derived in details in appendix A.2 from the optimality conditions of the contracting problem. This equation is similar to theorem 1 in Burdett and Coles (2003), lemma 3.2 in Shi (2009) and proposition 2 in Balke and Lamadon (2022), except that consumption c_t now replaces wages w_t on the left-hand side.

The intuition behind equation (5) is well understood in the literature. It equates the benefits from backloading wages on the right with the costs on the left. Backloading wages means that firms reduce w_{t-1} to increase w_t, which increases the continuation value at the current job W_t. The benefit of backloading wages is to reduce the worker EE separation rate. This benefit depends on the extend to which promising workers higher values will reduce the EE rate, captured by the term $p_W(W_t, a_t)$, and on the value that firms get from the match, captured by the term in parenthesis.

Backloading wages is costly because it generates a gap in the marginal utility of consumption of workers over time. When workers are hand-to-mouth, backloading wages implies backloading consumption because $c_t = w_t$ so the cost of backloading is straightforward and equation (5) is enough to characterize the optimal contract, together with the definition of the worker and firm values $W_t, \Pi(s_t)$. By contrast, when workers can trade risk-free bonds, consumption and wages are not always equal so equation (5) must be combined with a new optimality condition relating wages, consumption and assets. This new condition is the pseudo Euler equation that we derive next.

The pseudo Euler equation This equation is derived from the optimality condition for assets in the optimal contract. The surprising result is that this condition takes the familiar form of an Euler equation, except for an additional term that arises because assets influence directly the search decision of workers. This result is not obvious from inspection of the optimal contract (3) because the first order condition includes terms like $S_a(W_{t+1}, a_{t+1})$, which describes how the worker surplus from EE separations depends on assets. This term in turn depends on the fact that assets influence the search decision of workers $v(W_{t+1}, a_{t+1})$ and the matching rate in each market $\lambda_w(v_{t+1}, a_{t+1})$. We now manipulate this first order condition to show how it turns into the pseudo Euler equation from proposition 2, and then use this equation together with equation (5) to analyze how wages, consumption and assets change with tenure and over the life-cycle.

The first step to derive the pseudo Euler equation is to understand how assets influence job finding rates. For this, we turn to the free entry condition (4) that relates the value of new matches Π_0 and the vacancy filling rate λ_f (and thus the job finding rate λ_w through the matching function). First, notice that $\Pi_0(v, a, x_0) = \Pi_0(v, 0, x_0) + (1 + r)a$, so the value of new matches increases in the asset of the worker. Next, notice that the firm value Π_0 is strictly decreasing in the worker promised value v from the envelope condition. This suggests the existence of an indifference condition for firms between matching with a worker in a market with high value and high assets, and in a market with low value but also low assets. This also means that in equilibrium firms post relatively more vacancies in markets where workers ask for low values v, and in markets where they have high initial assets a. This relation is formally captured by the following equation, derived by combining the free entry condition with the envelope conditions,

$$\partial_a \lambda_w(v_{t+1}, a_{t+1}) = -\partial_v \lambda_w(v_{t+1}, a_{t+1})(1 + r)u'(c^{ee}_{t+1}) \tag{6}$$

where c^{ee}_{t+1} is the consumption of the worker at the next job after an EE separation. This equation states that, in equilibrium, increasing the assets of workers by 1% has the same effect on the EE probability than inducing workers to search in a market where the worker value is $(1 + r)u'(c^{EE}_{t+1})$% lower.

The next step is to combine this equation with the optimality condition for search v

$$\begin{aligned} S_a(W_{t+1}, a_{t+1}) &= \partial_a \lambda_w(v_{t+1}, a_{t+1})\left[v_{t+1} - W_{t+1}\right] \\ &= -\partial_v \lambda_w(v_{t+1}, a_{t+1})(1+r)u'(c^{ee}_{t+1})\left[v_{t+1} - W_{t+1}\right] \\ &= p(W_{t+1}, a_{t+1})(1+r)u'(c^{ee}_{t+1}) \end{aligned}$$

The first line uses the definition of $S(W, a)$ and the envelope theorem, the second line uses equation (6) and the third line uses the optimality condition for search v. Intuitively, raising the assets of workers increases their job finding rate so workers respond optimally by searching for jobs with a higher value, and thus a higher consumption at the next job. Thus, this equation shows that under the optimal contract increasing the worker's assets marginally by Δa has a similar effect on the worker value than increasing consumption at the next job by $(1 + r)\Delta a$, which is what an Euler equation would imply. This is why the optimality condition for assets a_{t+1} in the optimal contract can ultimately be written as a pseudo Euler equation.

Proposition 2. *The optimal contract satisfies a pseudo Euler equation*

$$\frac{u'(c_t)}{\beta(1+r)} \geq \delta u(c^u_{t+1}) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p_{t+1}u'(c^{ee}_{t+1}) + (1-p_{t+1})u'(c_{t+1})|x_0, x_t\right] - \mathcal{W}_t \qquad (7)$$

where $\mathcal{W}_t$ represents how firms use assets for worker retention, and is equal to

$$\begin{aligned} \mathcal{W}_t \equiv \;& (1-\delta)\frac{u'(c_t)}{\beta(1+r)}\mathbb{E}_{x_{t+1}}\Bigg[\left(u'(c_{t+1})p_W(W_{t+1}, a_{t+1}) + \frac{p_a(W_{t+1}, a_{t+1})}{1+r}\right) \\ & \times \left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_0, x_t\Bigg] \end{aligned}$$

Equation (7) holds with equality when the borrowing constraint (BC) does not bind.

Proof. See appendix A.3. □

Equation (7) can be separated into two groups of terms. The first group includes everything but $\mathcal{W}_t$ and constitutes a standard Euler equation. The second group of terms, gathered in $\mathcal{W}_t$, shows how firms use the worker's assets to manipulate the search decision, and therefore the worker EE rate. Quantitatively, virtually all the contract dynamics are driven by the first group of terms representing an Euler equation so I will focus the discussion on them and abstract from $\mathcal{W}_t$[15].

[15] $\mathcal{W}_t \neq 0$ because of well-known wealth effects on search, as in Acemoglu and Shimer (1999), and because the assets of workers influence how backloaded wages can be after an EE transition. The second effect is novel to optimal wage contracts with borrowing constraints but because the effect on contracts is so small I

Implications for tenure profiles We are now ready to combine equations (5) and (7) to analyze how assets influence the tenure profiles for wage and consumption.

Equation (7) states that, given a path for wages, the optimal contract will seek to use the worker's assets to smooth consumption over time within matches, but also across states after UE or EE separations. Consider for instance the case where wages are backloaded, which is generally the case in these models. Firms can replicate the allocation with hand-to-mouth workers and set $c_t = w_t$, but this would make consumption grow over time. As a result, contracts would be relatively unattractive to workers with concave utility who would prefer consumption to be stable over time. Firms would then have to pay workers higher average wages in order to attract them. Instead, firms could make the contract more attractive by depleting the worker's assets to smooth consumption over time. In equation (7), this means equalizing the terms $u'(c_t)/\beta(1+r)$ and the term $(1-\delta)\mathbb{E}_{x_{t+1}}\left[p_{t+1}u'(c^{ee}_{t+1}) + (1-p_{t+1})u'(c_{t+1})|x_0, x_t\right]$. However, firms would not want to deplete the worker's assets too much because workers also value precautionary savings against unemployment risk. In equation (7), this means equalizing the terms $u'(c_t)/\beta(1+r)$ and the term $\delta u(c^u_{t+1})$.

Consider now how firms set wages over time, knowing that they can smooth consumption using the worker's assets. When workers have plenty of existing assets and do not need to accumulate precautionary savings, firms know that they can smooth the worker's consumption using their existing assets despite wages being backloaded. As a result, they choose to backload wages even more to enhance worker retention. In the extreme, we recover case 2 from proposition 1 where assets are used to substitute for transfers post EE separations. By contrast, when workers do not have much existing assets and really want to accumulate precautionary savings, firms choose to backload wages less to help workers self-insure against unemployment risk. In the extreme, we recover case 3 from proposition 1 where assets are used to substitute for transfers post EU separations. Therefore, the degree of wage backloading depends on whether firms can use the worker's access to financial markets to smooth their consumption over time, or whether they want to help workers self insure against unemployment.

The worker's initial assets is critical precisely because it influence firms' ability to smooth consumption, and thus to backload wages. If the worker does not have much initial assets, firms cannot backload wages too much because the borrowing constraint

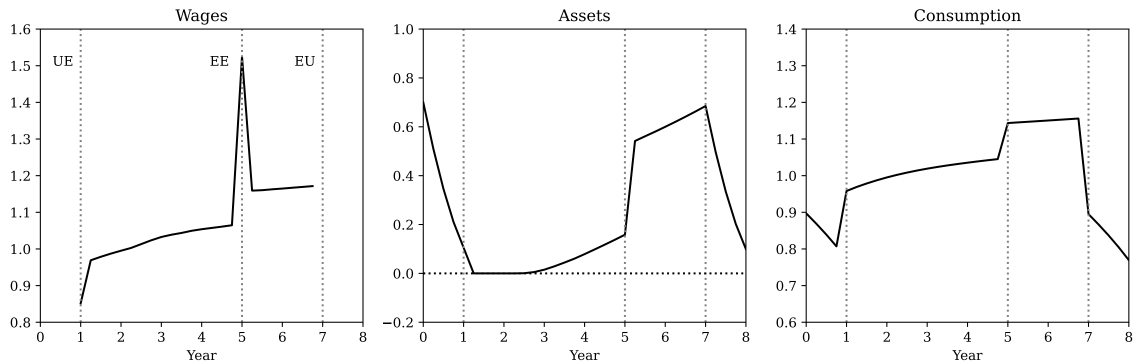
do not delve into it. In appendix A.5, I compute the optimal contract under the assumption that the savings decision a_{t+1} is private information. In this case, the optimal contract satisfies an Euler equation exactly. I find that the wage, consumption and asset dynamics are quantitatively indistinguishable from those of the model from section 2. This shows that the term $\mathcal{W}_t$ makes no difference quantitatively. Another way to see this is that when I simulate the model from section 2, I find that $\mathcal{W}_t/u'(c_t) \approx 0.001$ on average.

will bind. By contrast, if the worker has a lot of initial assets firms can backload wages more because the borrowing constraint is less likely to bind. Hence, a worker who has more assets to start with has access to a broad set of feasible contracts, and in general will search for different jobs than a worker with little assets. This mechanism generates some of the life-cycle dynamics from figure 2 and the differences in earning dynamics across the wealth distribution documented in section 4.2.

To summarize, this model can generate wages that are much more backloaded, but also much less relative to a model with hand-to-mouth workers. In fact, wages can even be frontloaded so a new implication is that firms sometimes offer hiring bonuses to workers. Furthermore, the extend to which wages are backloaded depends on the amount of uninsurable risk that workers face, and on their ability to smooth consumption using their existing assets or through borrowing. In section 4, I show that these implications are consistent with empirical evidence and that they matter for public policies that relax the borrowing constraint of workers.

Life-cycle dynamics Figure 2 illustrates the implications of the optimal contract for the life-cycle dynamics of wages, assets and consumption. In this specific example, the worker starts unemployed at $t = 0$ with existing assets $a_0 = 0.7$. The worker finds a job at year 1, makes an EE separation at year 5 and an EU separation at year 7. The left panel shows that wages increase during the first job between years 1 and 5. The worker then receives a hiring bonus when switching jobs, and wages mildly increase later on during the second job between years 5 and 7. The middle panel shows that the worker depletes assets when unemployed and accumulates assets when employed. Finally, the right panel shows that consumption falls while the worker is unemployment and rises while the worker is employed. Consumption jumps during UE, EE and EU transitions and is smooth otherwise.

Figure 2: Life-cycle dynamics for wages, consumption and assets



Note: example of paths for wages, assets and consumption for a worker who is unemployed with $a_0 = 0.7$ at $t = 0$. The dotted vertical lines show when the worker experiences UE, EE and EU separations. The paths are computed in the quantitative model where productivity is heterogeneous across firms and shocks are expected to occur but where the realization of productivity shock ν_t happen to be null.

What accounts for the life-cycle dynamics in figure 2, especially those between year 1 and 7 when the worker is employed? Given the path of wages, the paths of assets and consumption satisfy the pseudo Euler equation (7), meaning that workers smooth consumption over time and state. The path for wages reveals how firms balance their desire to retain workers with their desire to insure them. Specifically, wages are backloaded during the first job. In fact, wages are so backloaded that the worker does not accumulate precautionary savings at all during approximately two years and consumes her entire savings during the first period of employment. If workers could borrow, firms would backload wages even more. The reason for this backloading of wages is that workers hired from unemployment are at the bottom of the job ladder. As a result, they receive a low wage so the firm value from the match is high. Besides, the EE separation rate of these workers is high so backloading their wages leads to a large reduction in their quit rate ($p_W(W, a)$ is large). This implies that the benefits of backloading wages on the right of equation (5) is large. Furthermore, the income of this worker does not fall too much if they become unemployed because the wage is low so the worker does not want to accumulate much precautionary savings. This means that the cost of backloading wages on the left of equation (5) is small. These observations imply that the retention motive is stronger than the precautionary savings motive so firms choose to backload wages significantly. The backloading of wages means that workers are initially hired with low wages but promised higher wages in the future, so the wage of workers increases over time. As a result, the strengths of the retention and insurance motives change over time. Around year 3, firms decide to backload wages slightly less in order to help workers accumulate precautionary savings. At this point, firms still get a positive value from the match and therefore want to retain workers but they are willing to let workers accumulate precautionary savings nevertheless. At year 5, the worker finds another job and receives a hiring bonus from the new firm, which leads to the jump in consumption and precautionary savings. This new firm also wants to retain its worker but the precautionary savings motive is now much stronger so new firms choose to attract the worker by offering insurance against unemployment risk in the form of frontloaded wages.

In conclusion, these results show that letting workers trade risk-free bonds leads to much richer life-cycle dynamics for wages, consumption and assets. We explore some of these implications with the quantitative model in section 4.

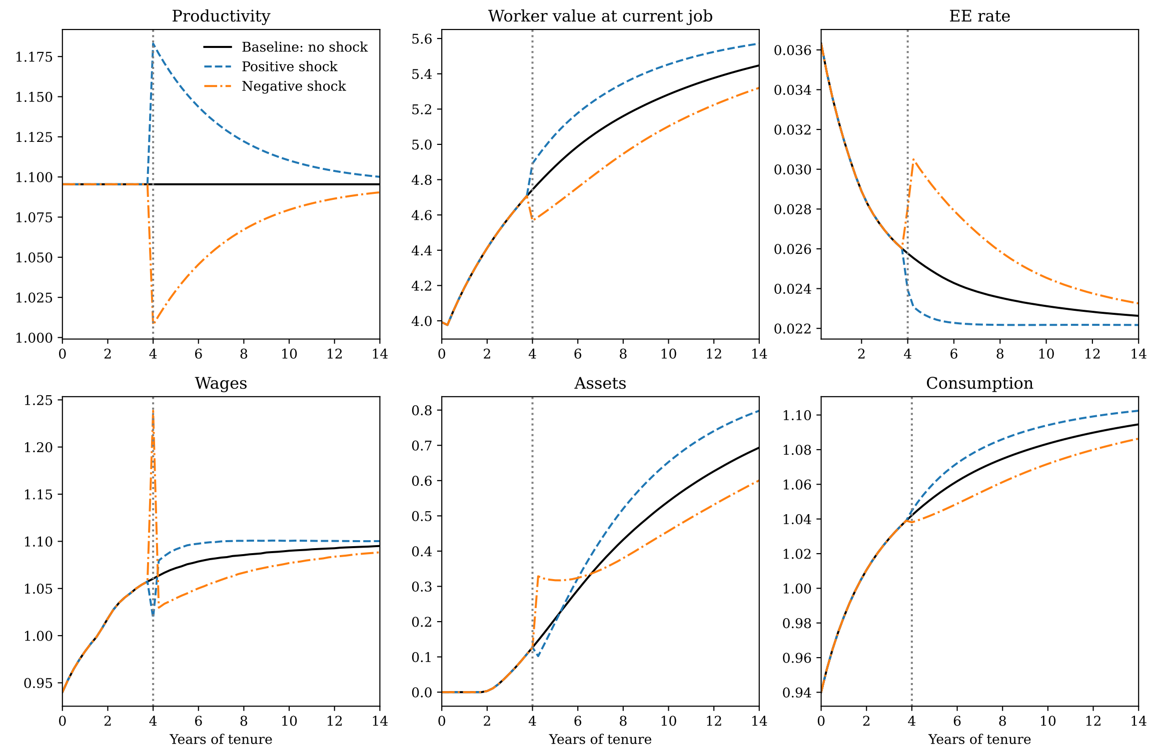
3.3 Implications for the pass-through of productivity shocks

This section describes how firms pass productivity shocks through to the wage, assets and consumption of workers. I focus here on describing how letting workers trade risk-free bonds influences the *dynamics* of the pass-through whereas section 4 will show that it also influences its *size*.

Figure 3 compares three scenarios for a worker who is hired from unemployment at $t = 0$ with no initial asset and happens to remain employed with the same firm throughout, in the quantitative model. In the first scenario described by the solid black line, productivity happens to remain constant throughout the match. In the second scenario described by the dashed blue line, productivity jumps up by one standard deviation after 4 years of tenure and then reverts back to its average level according to its persistence ρ estimated in the data. In the third scenario described by the dash-dotted orange line, productivity jumps down by one standard deviation after 4 years of tenure and then reverts back. The pass-through of a positive productivity shock is defined in this section as the difference between the dashed blue line from the second scenario and the solid black line from the first scenario. The pass-through of negative shocks is defined similarly.

Consider first the top panel of figure 3, which shows the paths of productivity x_t, of the worker continuation value at the current job W_t and of the worker EE expected separation rate p_t. After a positive shock to productivity, the firm chooses to increase the worker value at the current job in order to reduce the EE rate. Conversely, after a negative shock the firm chooses to reduce the worker value to increase the EE rate. The intuition behind this pass-through is similar to models with hand-to-mouth workers (e.g. see Souchier, 2023). Firms face a trade-off between insuring workers against risk and optimizing worker retention. On one hand, firms seek to insure workers against risk by keeping their consumption, and therefore their continuation value, independent of productivity shocks. On the other hand, firms seek to optimize workers retention by paying workers relatively more when productivity, and therefore profits, is high and relatively less when productivity is low. The optimal pass-through thus balances the worker's demand for insurance and the firm's desire to optimize worker retention.

Figure 3: The pass-through of productivity shocks to workers



Note: the top panel shows the paths for productivity x_t, worker value W_t and expected EE rate p_t for a worker hired at $t = 0$ from unemployment with zero assets. The bottom panel shows the paths of wages, assets and consumption. The paths are computed in the quantitative model where productivity is heterogeneous across firms and shocks are expected to occur. The dotted vertical line at 4 years shows when the productivity shock ν_t actually occurs. The solid black line represents the paths when the realization of productivity shocks ν_t happens to be null, the dashed orange line the paths when the realization of productivity is positive and the dash-dotted orange line when the realization is negative. Throughout, EE and EU shocks happen to be zero so the worker remains employed at the current firm.

The novel result here, and perhaps the most surprising, is how firms choose to implement this pass-through. This is shown in the bottom panel of figure 3, which shows the pass-through to wages w_t, assets a_t and consumption c_t. Consider the pass-through of *positive* shocks. The left panel shows that firms cut wages on impact and then increase wages substantially in the future. Meanwhile, workers deplete temporarily their assets to increase their consumption smoothly over time. Therefore, firms choose to increase the worker value after the shock with promises of future wage increases. Why would firm adopt such a strategy? Remember from section 3.2 that the path of wages over time depends on a trade-off between worker retention and insurance against unemployment risk. After a positive shock to productivity, the retention motive becomes stronger so firms choose to backload wages more, even if it means that workers are less insure against unemployment risk temporarily. In fact, firms backload wages so much that they fall on impact as in figure 3. Thus, after the shock firms seek to retain workers more not only by paying them more on average, but also by backloading their wages more. The persistence of shocks is critical for this result. If shocks were temporary, wages would instead increase only on impact. This is because backloading wages helps to retain workers in the future, which is only valuable if shocks are sufficiently persistent. In comparison, the model with hand-to-mouth workers does not feature these rich dynamics because firms do not change the degree of wage backloading in response to shocks.

The dynamics shown in figure 3 are quite stark, and not necessarily very realistic. Indeed, we do not usually see wages fall after positive productivity shocks in the data. These stark predictions however are due to the stylized nature of the model. For example, in a richer environment where firms have imperfect information about the persistence of shocks and respond to shocks as if they are temporary, wages smoothly rise over time in response to positive shocks[16]. Despite its stylized nature, the model from section 2 still sheds new light on existing empirical evidence. For instance, the model with assets is consistent with evidence that only persistent productivity shocks have persistent effects on wages whereas temporary shocks do not (Chan, Salgado and Xu, 2025). By contrast, even temporary shocks have persistent effects in models with hand-to-mouth workers because firms seek to smooth consumption over time. In section 4, I use the estimated model to compare the size of the pass-through for workers across the wealth distribution. Because of the complex dynamics shown in figure 3, I will focus in this exercise on the cumulative pass-through to wages and consumption because it is less sensitive to the exact timing of wage changes.

[16]Other factors could make wages less likely to fall on impact after positive shocks. For example, in a model with a life-cycle firms might not choose to backload wage increases so much for older workers because they are about to retire, and for young workers because they have little assets. Similarly, in a model with financial frictions firms might not choose to backload wage increases so much because this is precisely when their financial constraint binds less.

4 Quantitative implications

The previous section showed that the assets of workers influence the wage contract they receive from firms. In this section, I investigate further the implications of this prediction. To this end, I first estimate the model using matched employer-employee data from France. I then show that wealthy workers receive wages that are more backloaded and have a higher pass-through relative to poor workers, consistent with existing empirical evidence. Finally, I show that this has important implications for policies relaxing the worker's borrowing constraint. This policy improves not only the insurance that workers receive, as in models with exogenous contracts, but also allocative efficiency.

4.1 Estimation

I estimate the model at quarterly frequency using administrative and survey data on labor income, worker mobility, assets and firm productivity from France.

Data The main data used in this paper is French matched employer-employee data that contains information on wages, worker mobility and firm productivity. The data covers 1/12th of the French labor force and the universe of firms between 2009 and 2019. I focus on prime age workers (25-55 years old) who work in the private sector in for-profit firms with at least 3 employees, and who have a full-time job with a permanent contract. The sample contains approximately 530,000 workers and 130,000 firms per year. I supplement this data with information on liquid assets from the Household Finance and Consumption Survey and information on unemployment benefits from the OECD.

Estimation I estimate the model in two steps: first, I set some parameters externally; second, I infer the remaining model parameters by moment matching.

The model parameters set externally are the utility function, the foreign interest rate r and the matching function. The utility function is CRRA with coefficient $\gamma = 2$, following standard estimates from the macroeconomic literature. I set the interest rate to 1% quarterly. The matching function is Cobb-Douglas

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu} \phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). $B = 0.26$ is calibrated to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the job finding rate in my model. The cost of posting vacancy is modeled as

$$k(x_0) = k \exp\left(\phi(x_0 - 1)\right)$$

with $\phi = 10$.

The other model parameters are inferred by matching moments in the French data and in model-simulated data. Specifically, I simulate a panel of workers in the model and estimate the exact same set of moments in the model and in the data. The estimated parameters are the discount rate β, the vacancy posting cost k, the search efficiency on the job κ, the value of home production b, the exogenous separation rate into unemployment δ, the persistence of productivity ρ_x, its volatility σ_x and the volatility of i.i.d. measurement errors for annual productivity σ_x^{meas}. These 8 parameters are estimated using 8 moments in the data.

Table 1 shows the moments used in the estimation and the parameters. I use estimates of labor market flows (UE, EE and EU rates), of the replacement ratio for unemployment benefits and of liquid assets relative to labor income to discipline the mobility of workers across jobs, the risk that workers face and their access to financial markets. I use moments on firm productivity, measured as value added per worker, to discipline the risk that firms face and might transmit to workers. The UE, EE and EU rates are small relative to existing estimates in the literature, but this is not surprising given that they are measured for France and for workers with strong ties to the labor market. The replacement ratio for unemployment benefits is measured using OECD data, and is higher than existing estimates for the United States (e.g. Chodorow-Reich and Karabarbounis, 2016). The median liquid assets relative to labor earnings is measured using estimates for France from the Household Finance and Consumption Survey. Appendix X provides details on the computation of these moments.

The model is jointly estimated so all moments influence all parameters but the mapping between moments and parameters displayed in table 1 nevertheless gives some intuition about the estimation. In particular the amount of liquid asset is critical in pinning down the discount factor of workers relative to the interest rate $\beta(1+r)$. This new moment is standard in the precautionary savings literature (see Auclert, Rognlie and Straub, 2024), but novel for dynamic contract. If $\beta(1+r) = 1$ workers would accumulate an infinite amount of savings (as in Sotomayor, 1984, Chamberlain and Wilson, 2000) whereas when $\beta(1+r)$ is small, workers are relatively impatient and do not accumulate too much assets. A novel implication of this assumption in the context of optimal contracts is that it makes the promised value of workers drift over time while employed. This can be

Moments	Data	Model	Parameters	
Quarterly UE rate	21%	21.7%	Cost of posting vacancy k	0.2
Annual EE rate	6.3%	6.3%	On-the-job search efficiency κ	0.45
Annual EU rate	7%	7.0%	Separation rate δ	0.0195
Replacement ratio for unemp. benefits	62%	63%	Flow unemployment value b	0.7
Liquid assets/annual labor earnings	16.8%	16.5%	Relative discount factor $\beta(1+r)$	0.994
Variance of productivity growth	0.04	0.047	Volatility of productivity σ	0.08
Autocorrelation of order 1	- 0.22	- 0.24	Persistence of productivity ρ	0.93
Autocorrelation of order 2	- 0.06	- 0.068	Volatility of meas. error σ^{meas}	0.22

Note: the left panel shows the moments used in the estimation in the data and in the model. The right panel shows the parameters estimated internally. The UE, EE and EU rates and the moments on productivity are calculated using the French matched employer-employee data. The replacement ratio is calculated using data from the OECD on unemployment benefits. The liquid assets/annual labor earnings ratio is calculated using data from the Household Finance and Consumption Survey.

Table 1: Targeted moments in data vs. model and parameters

seen from equation (5) when workers cannot switch jobs ($\kappa = 0$), as the consumption of workers falls over time when $\beta(1+r) < 1$ even in the absence of EE separations.

Implications for contracts Table 2 shows key moment on wage contracts from the estimated model with assets and from a model with HtM workers. In the model with HtM workers, all parameters are kept constant except that $a_{t+1} = 0$ for employed and unemployed workers. The moments from table 2 are calculated from a panel of workers in model-simulated data. The wage and consumption growth are measured as the annual growth rate of wage and consumption for continuously employed workers. The pass-through of productivity shocks to wages and consumption are measured as the OLS coefficient of annual productivity growth on cumulative earnings growth over 2 years[17].

The first column of table 2 shows that wages grow on average by 1.9% per year in the model with assets and only by 0.6% per year in the model with HtM workers. This difference arises because firms take advantage of worker's existing assets to backload wages more in order to enhance worker retention. There are instances where firms choose to backload wages less in the model with assets to help workers self-insure (for example after EE separations), as explained in section 3.2, but on average wages are more backloaded when workers can trade risk free bonds. Workers use their assets to smooth their

[17]Specifically, I first compute the annual productivity and wage as the average within a firm-worker match across quarters, denoted $\overline{x}_y$ and $\overline{w}_y$ for year y. I then estimate the pass-through as

$$\theta^{w,x} \equiv \frac{\text{Cov}(\Delta \log \overline{x}_y, \Delta \log \overline{w}_y + \Delta \log \overline{w}_{y+1})}{\text{Var}(\Delta \log \overline{x}_y)} \tag{8}$$

The consumption pass-through $\theta^{c,x}$ is defined similarly.

consumption over time so their consumption is more stable in the model with assets than with HtM workers (0.5% vs 0.6%), as shown in the second column.

The third column from table 2 shows that after a 10% increase in annual productivity, wages increase by an average of 5.4% in the next 2 years in the model with assets. By contrast, in the model with HtM workers, wages increase by only 1.29% so about 4 times less. This means that workers receive much less insurance from firms when they can trade risk-free bonds than when they are HtM. However, despite receive less insurance from firms, workers receive more insurance overall when they can trade risk-free bonds. This can be seen by looking at column 4, which shows that consumption responds by 0.88% after a 10% increase in productivity in the model with assets, and by 1.29% in the model with HtM workers. The reason for this difference is that firms do not need to smooth wages across states when workers can smooth consumption themselves using their existing assets so they pass productivity shocks through relatively more to optimize worker retention. The marginal propensity to consume implied by the model can be computed as the ratio of the consumption pass-through to the wage pass-through. It is equal to $8.8/54 = 16\%$ in the model with assets and $11.9/11.9 = 100\%$ in the model with HtM workers, which highlights that workers receive significant insurance outside firms against productivity shocks.

Taken together, these results show that workers use financial markets to smooth considerably their consumption over time and across states. In turns, the insurance that workers receive outside firms significantly crowds out the insurance that workers receive inside firms in the sense that firms smooth wages much less over time and across states when workers have access to financial markets[18]. The fact that this crowding out is large is one of the main lessons from the quantitative exercise, and it will also be at work when we compare wage contracts across the wealth distribution in sections 4.2 and 4.3 and when we relax borrowing constraints in section 4.4.

Validation The model is broadly consistent with empirical evidence on wage growth and pass-through, which were not targeted in the estimation. In particular, the tenure profile for wages, which measures the degree of wage backloading, is almost identical in the model (18%) and in the data (14%)[19]. My estimate for the pass-through of productivity

[18] To illustrate the size of the crowding out, consider the following back-of-envelope calculation: if letting workers trade risk-free bonds did not influence wage contracts, the pass-through would have remained equal to 11.9% in the model with assets; the pass-through to consumption would therefore have been approximately equal to $11.9\% \times \text{MPC} = 11.9\% \times 16\% = 1.9\%$, which is less than a quarter the pass-through in the model with assets (8.8%). The same back-of-envelope calculation can be done for wage growth.

[19] The tenure profile is measured as the cumulative wage growth after 25 years of tenure at the same firm. In the data, the tenure profile is measured after controlling for overall experience in the labor market using

	Growth w	Growth c	Pass-through to w	Pass-through to c
Model with assets	1.9%	0.5%	54%	8.8%
Model with HtM workers	0.6%	0.6%	11.9%	11.9%

Note: the top row reports moments computed in the model with assets and the bottom row reports the same moments in a model with hand-to-mouth workers. The model with HtM workers is calibrated using exactly the same parameters as the baseline model, except that the constraint $a_{t+1} = 0$ is imposed for unemployed and employed workers. The wage and consumption growth are calculated annually for continuously employed workers. The pass-through to wages and consumption are computed as the regression coefficients of 2-year cumulative wage and consumption growth on annual productivity growth.

Table 2: Implications for wage contracts

shocks to wages is between those typically measured using administrative data and statistical models of earnings (e.g. Guiso et al., 2005) and those measured using exogenous shocks to firm productivity (e.g. Kline, Petkova, Williams and Zidar, 2019)[20]. In sections 4.2 and 4.3, I show that the model is also consistent with existing empirical evidence that wage growth and wage pass-through vary across the wealth distribution. Finally, the model-implied marginal propensity to consume ($8.8/54 = 16\%$) is much smaller than 1 but larger than 0, consistent with existing empirical evidence.

4.2 Wealth at birth and life-cycle earnings inequality

I use the model to evaluate how the optimal degree of *wage backloading* depends on the worker's assets, and show that workers born rich select jobs where wages are more backloaded but also pay more on average.

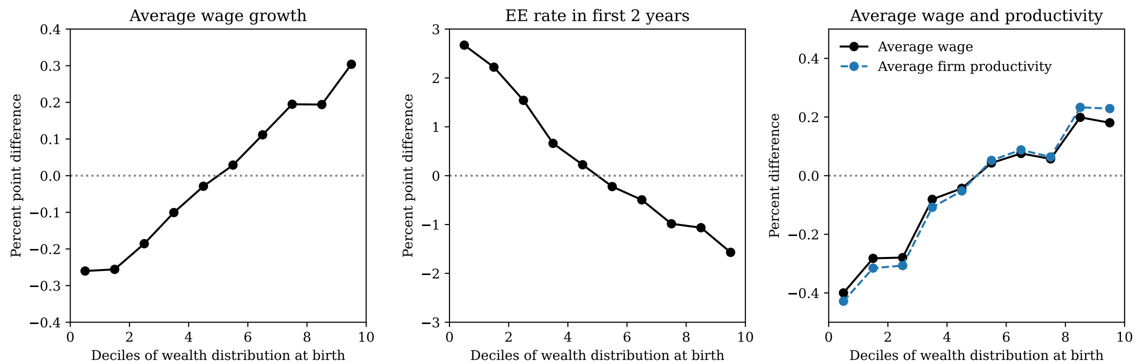
Figure 4 shows how the average wage growth, the EE rate and the average wage and productivity depend on the worker's wealth "at birth". To compute this figure, I simulate the career of a panel of workers starting unemployed with levels of assets corresponding to different deciles of the wealth distribution[21]. The asset workers hold at the start of the simulation is their wealth "at birth". This exercise captures, in reduced form, the different trajectory of workers who start their career with different initial assets and therefore experience different life-cycle income profiles. The figures are normalized so the median equals 0.

a polynomial in experience.

[20]Estimates using statistics model of earnings typically range between 3% and 12% across countries but these are subject to downward bias due to measurement errors. Estimates using identified shocks are typically much larger but these estimates are measured in specific markets not representative of my sample. For example, Kline et al. (2019) study the market of patent inventors in the United States.

[21]I assume that workers find a job in the first period to better emphasize the impact of initial asset holdings on wage contracts.

Figure 4: Wealth at birth and life-cycle earnings inequality



Note: the statistics are computed by simulating a panel of workers starting from unemployment at $t = 0$ with wealth drawn from the stationary distribution. The wealth that workers have at the start of the distribution is their "wealth at birth". The left panel shows the average annual growth rate of wages, the middle panel shows the average EE rate of workers during the first 2 years after they leave unemployment and the right panel shows the average wage of workers and average productivity of matches. The figures are normalized so that the median equals 0.

The left panel shows that relatively wealthy workers, who start their career with more assets, experience higher wage growth than relatively poor workers, who start their career with less assets (e.g. the top decile experiences an average annual wage growth of 1.4% compared to 0.9% for the bottom decile, a difference of 0.5 percent points). The reason is that wealthy workers receive wages that are more backloaded so their wages growth faster on the job. The consumption of wealthy workers, however, is more stable over time than the consumption of poor workers (not shown in figure 4). The middle panel shows the cumulative EE rate within the first 2 years since workers first match with their employers. The rate is 10.7% for workers at the bottom decile as opposed to 7.1% for workers at the top decile, a difference of 3.6 percent points. The reason why relatively rich workers are less likely to switch jobs is precisely because their wages are more backloaded. Finally, the right panel shows the average wage in solid black and average productivity in dotted blue for workers across the wealth distribution. Relatively rich workers receive average wages 0.55% higher than poor workers, and they are matched with firms that are 0.65% more productive on average. The reason is that firms' expected profits increase when they are more likely to retain workers. Because of the free entry condition, these firms offer workers higher average wages. Besides, firms are more likely to invest in better technology x_0 because they are less concerned about losing their workers. Another way to say this is that the hold-up problem that prevents firms from investing optimally in productivity x_0 is lessened for wealthy workers because these workers effectively pay the upfront investment in technology x_0 themselves with backloaded wages. Thus, income and productivity inequality here arise because poor workers are further away from the first-best level of investment than wealthy workers are. Taken together, figure 4 shows that wealthy workers select jobs where wages are more backloaded but that pay more on average in equilibrium. As a result, firms know that they can retain these workers better and choose to invest more in productivity. By contrast, poor workers select jobs with more stable earnings because they cannot smooth consumption themselves but these jobs also pay less and have lower productivity[22].

This mechanism is consistent with evidence on earnings growth across workers documented in the literature. In particular, Guiso et al. (2012) find using Italian data that firms that become financial constrained are more likely to backload the wages of workers who are more likely to be wealthy, namely managers and white-collar workers relative to blue-collar workers. Halvorsen et al. (2022) use administrative data from Norway and find that the children of parents with high net wealth experience higher wage growth than children of parents with low net wealth. The model also provides a new explanation for the widely documented persistence of income inequality across generations (e.g. Solon, 1992) in that parents with high earnings are more likely to be wealthy, thus allowing their kids to select jobs with wages that are more backloaded but also higher on average.

4.3 Insurance against productivity shocks over the wealth distribution

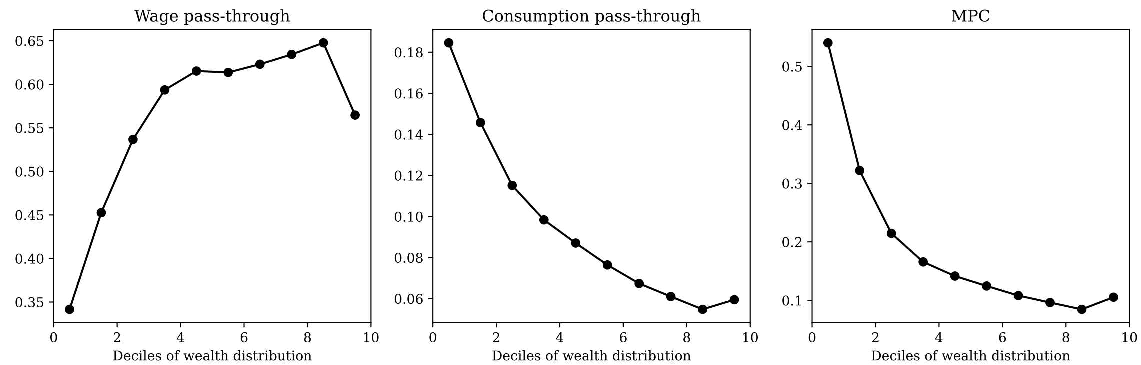
I use the model to evaluate how the optimal degree of *pass-through* depends on the worker's assets, and show that wealthy workers receive less insurance from firms but more insurance overall against these shocks.

Figure 5 shows the pass-through, defined in equation (8), for different quantiles of assets in the previous year. To compute this figure, I simulate a panel of workers and compute the pass-through in the stationary distribution. The left panel shows that the pass-through to wages increases over the wealth distribution, except for the top decile. The middle panel shows that the pass-through to consumption has the opposite pattern and is larger for relatively poor workers. Finally, the right panel shows that the implied marginal propensity to consume, defined as the ratio of the pass-through to consumption over the pass-through to wages, falls with assets. The results from figure 5 thus show

[22] This mechanism complements the one described in Eeckhout and Sepahsalari (2023) and Chaumont and Shi (2022) with directed search and fixed-wage contracts. In these papers, wealthy workers receive higher wages on average because they choose to search in markets with a lower job finding rate but higher average wages and productivity. This mechanism is also at play in my model but it is reinforced by the ability of firms to backload wages. A key difference is that models with fixed-wage contracts imply that poor workers experience relatively high wage growth relative to wealthy workers because they start from the bottom of the job ladder. By contrast, with optimal contracts wealthy workers are the ones who experience higher wage growth as suggested in the data.

that workers along the wealth distribution receive different mix of insurance inside and outside the firm. Relatively poor workers receive more insurance inside the firm (their wage pass-through is low and their MPC is high) whereas relatively rich workers receive more insurance outside the firm (their wage pass-through is high but their MPC is low).

Figure 5: Heterogeneous pass-through over the wealth distribution



Note: the statistics are computed for each bin of the wealth distribution from the previous year. The pass-through to wages and consumption are computed as in table 2. The MPC is calculated as the ratio of the pass-through to wages to the pass-through to consumption.

Why won't firms insure relatively poor workers even more? The reason is that relatively poor workers tend to be at the bottom of the job ladder. As a result, they are more likely to switch jobs and receive lower wages from firms. Firms thus find it optimal to increase their consumption relatively more in response to positive productivity shocks to induce them to stay, and reduce their consumption relatively more in response to negative productivity shocks to let them go. By contrast, firms are willing to provide more insurance overall to rich workers because they are less likely to switch jobs and they generate less profit to firms.

The results from figure 5 are consistent with existing evidence from the literature on pass-through and marginal propensity to consume. Specifically, Fagereng, Guiso and Pistaferri (2017) show that the pass-through of firm-level productivity shocks to wages is increasing in assets using data from Norway. Remarkably, in their sample the pass-through is about twice larger for the top decile than it is for the bottom decile, which is consistent with figure 5. Finally, the fact that the marginal propensity to consume is decreasing with liquid assets is also consistent with a broad set of evidence.

Experience < 10 years	**Growth** w	**Growth** c	**Pass-through** w	**Pass-through** c	**Avg.** w	**Avg.** x
Baseline ($\underline{a} = 0$)	1.2%	1.4%	48%	14%	1.089	1.18
Counterfactual ($\underline{a} = -2$)	4.9%	0.4%	68%	10%	1.096	1.20

Note: the top row considers the baseline model without borrowing whereas the bottom row considers a model calibrated with the same parameters except that workers can borrow up to $\underline{a} = -2$, which corresponds to about 2 quarters of average labor income. The statistics are computed for workers with less than 10 years of experience, which means that they were unemployed with no asset less than 10 years ago. The first two columns report the average growth rate of wages and consumption. The next two columns report the pass-through of wages and consumption, computed as in table 2. The last two columns report the average wage of workers and the average productivity of matches.

Table 3: Implications of relaxing borrowing constraints

4.4 Public insurance policies: relaxing the borrowing constraint

The previous sections showed that workers across the wealth distribution receive different amounts of insurance inside and outside firms. In this section, we show that policies that relax borrowing constraints have a similar effect in that it improves the insurance that workers receive outside firms. As a result, workers who can now borrow receive less insurance from firms, but more insurance overall, and they end up being better matched with firms and receive higher average wages. In this sense, letting poor workers borrow makes them similar to wealthy workers.

Table 3 presents the result of this counterfactual exercise. The first line shows the baseline model where workers cannot borrow whereas the second line shows the counterfactual economy where workers can can borrow at rate r up to -2, which corresponds to approximately 2 quarters of labor earnings. I interpret this counterfactual as arising from a policy improving worker's access to borrowing, such as subsidizing loans to workers. In both the baseline and the counterfactual, I focus on workers with less than 10 years of labor market experience, meaning workers who were unemployed with 0 assets less than 10 years ago[23]. I focus on this group because workers with more labor market experience have the time to accumulate precautionary savings and are thus less affected by this policy. In the counterfactual economy, all the model parameters are kept constant except the borrowing constraint.

Table 3 shows that workers receive less insurance from firms but more insurance overall when the borrowing constraint is relaxed. This can be seen along 2 dimensions. First, the growth of wages is much higher (4.9% compared to 1.2%) but the growth rate of consumption is much lower (0.4% compared to 1.4%). The reason is that workers select jobs

[23]Focusing on workers who start from unemployment with 0 assets is a parsimonious way of capturing the life-cycle of workers without having to model it explicitly.

with backloaded wages when they can borrow, and then use their ability to borrow to smooth their consumption. Second, the pass-through of productivity shocks to wages is much higher (68% compared to 48%) but the pass-through to consumption is much lower (10% compared to 14%). The reason is that workers select jobs with more volatile earnings when they can borrow because they can smooth their consumption by borrowing. In fact, the implied marginal propensity to consume falls from 28% to 14% when workers can borrow. Overall, the crowding out of the insurance inside the firm by the insurance outside the firm is large, as in table 2. The most surprising result from table 3 is that workers are matched with more productive firms (1.2 compared to 1.18) and receive higher average wages (1.096 compared to 1.089) when the borrowing constraint is relaxed. The reason is that allowing workers to borrow enables them to search for jobs that have more backloaded wages, as in section 4.2. Firms invest in better technology in these jobs because they are less worried about loosing their workers to competing firms. As a result, workers receive higher wages on average.

In conclusion, relaxing borrowing constraints improves the insurance that workers receive overall and improve matching efficiency. Workers however should expect to receive wages that are more backloaded and more volatile after this policy is implemented. Conducting this counterfactual exercise in a precautionary savings model without optimal contract would lead to overstate the benefits in terms of insurance, but understate the benefits in terms of allocative efficiency.

5 Hidden initial assets

So far I have assumed that the existing assets of newly hired workers a_0 are public information to firms. This assumption is important for two reasons. First, the initial level of assets a_0 determines worker's ability to smooth their consumption when their wages is backloaded and therefore the optimal contract that firms offer to workers. Second, firms need to know the worker's initial assets to compute how much value they can generate from a match. This is because it is cheaper to provide a higher value v to a worker with high initial assets a_0 and because workers with different initial assets a_0 might experience different EE transition rates during the match.

One justification for the assumption that initial assets a_0 are public information is that firms can proxy the worker's assets a_0 given the information they have about workers. For example, firms know about the education and occupation of workers, both of which are strongly correlated with wealth. Besides, firms typically know how long a worker has been unemployed before meeting with a firm. Firms can then design wage contracts given this information, for example by offering relatively more backloaded wage contracts to workers in wealthier occupations. However, even within occupations there is still significant heterogeneity in wealth across workers. A concern is therefore that the optimal contract derived in this paper might not be robust to this residual uncertainty about the worker's wealth. In this section, I argue that the contract is in fact robust for two reasons. First, workers voluntarily select contracts with specific degrees of risk depending on their initial assets. I illustrate this mechanism by focusing on a special case where the equilibrium in which assets a_0 is private information coincides with the equilibrium in which it is observable to firms, and yet workers receive very different wage contracts across the asset distribution. Second, even under conditions in which workers would not reveal their initial assets to firms truthfully, there exists a separating equilibrium in which firms modify optimal contracts slightly to induce truthful reporting of assets a_0 from workers. I derive this result by computing the equilibrium with adverse selection using the methodology of Guerrieri et al. (2010).

Model with adverse selection To evaluate the implications of private information about the worker's initial assets a_0, we need to consider a version of the model presented in section 2 in which the savings decision of employed workers is also private information. Otherwise, the only deviation workers could follow is to misreport their assets and consume more or less in the first period of employment. The model is therefore one where moral hazard arises as a result of two hidden actions (search v and savings a_{t+1}) and where adverse selection arises because the worker's initial assets a_0 is private information. To keep the problem tractable, I consider a 2-period version of the model ($t = 0, 1$) that can be solved numerically without recursive methods[24]. To further simplify notation, I assume that the match initial productivity x_0 is fixed to 1, that the interest rate and discount factor satisfy $\beta = 1 + r = 1$, that the matching function implies $\lambda = 1/\lambda_f$ and that a worker who does not find a job at $t = 0$ remains unemployed at $t = 1$ so that his continuation value is $2u(b + a_0/2)$.

In section 2, labor markets were indexed by the worker value v and their existing asset a. In the 2-period model, labor markets at $t = 1$ can instead be indexed by the wage that workers receive w^{ee}. The reason is that workers can impute the value v given the wage w^{ee} and the asset a and firms only need to know the wage w^{ee} to evaluate the profit from matching with a worker. This notation simplifies the analysis below. The free entry condition at $t = 1$ pins down the job finding rate $\lambda_1(w^{ee}) = (1 - w^{ee})/k$.

[24] Computing the full dynamic model with adverse selection is very challenging and not necessary to derive the results from this section. In comparison, Guerrieri et al. (2010) consider a static environment without moral hazard.

Consider now the search decision of an employed worker with promised wage w_1 at the current job and asset a. The optimality condition is

$$\lambda_1'(w^{\text{ee}})\left(u(w^{\text{ee}}+a)-u(w_1+a)\right)+\lambda_1(w^{\text{ee}})u'(w^{\text{ee}}+a)=0 \tag{9}$$

Denote $w^{ee}(w_1,a)$ the optimal search decision and define $p(w_1,a)\equiv\kappa\lambda_1(w^{ee}(w_1,a))$ and $S(w_1,a)\equiv p(w_1,a)\left(u(w^{\text{ee}}(w_1,a)+a)-u(w_1+a)\right)$.

Now consider the problem of a newly employed worker at $t=0$ with initial asset a_0, who saves a and receives a wage contract $w_0, w_1(x)$. This worker gets the value

$$V_0(a_0,a,w_0,w_1(x))\equiv u(w_0+a_0-a)+\delta u(b+a)+(1-\delta)\mathbb{E}_x\left[u(w_1(x)+a)+S(w_1(x),a)\right]$$

We can use this function to write the optimal contract at $t=0$ as the result of the following optimization problem, as in Moen (1997),

$$U_0(a_0)=\max_{a,\lambda_0,w_0,w_1(x)}\quad\lambda_0V_0(a_0,a,w_0,w_1(x))+(1-\lambda_0)2u(b+a_0/2) \tag{10}$$

subject to

$$\begin{aligned}
\text{(FE)}: &\quad k\lambda_0=1-w_0+(1-\delta)\mathbb{E}_x\left[(1-p(w_1(x),a))(x-w_1(x))\right]\\
\text{(Euler)}: &\quad u'(w_0+a_0-a)=\delta u'(b+a)+(1-\delta)\mathbb{E}_x\left[u'(w_1(x)+a)+S_a(w_1(x),a)\right]\\
\text{(IC-}a_0\text{)}: &\quad U_0(\hat{a}_0)\geq\max\nolimits_{\hat{a}\geq0}\lambda_0V(\hat{a}_0,\hat{a},w_0,w_1(x))+(1-\lambda_0)2u(b+\hat{a}_0/2)\quad\forall\hat{a}_0\neq a_0\\
\text{(BC)}: &\quad a\geq0
\end{aligned}$$

$U_0(a_0)$ denotes the value achieved by an unemployed workers at $t=0$ with initial asset a_0, a denotes the savings decision of the worker conditional on finding a job and λ_0 denotes the job finding rate of an unemployed worker at $t=0$. The first constraint (FE) is the free entry condition at $t=0$, the second (Euler) is the incentive compatibility constraint that must be satisfied because the savings decision of employed workers a is private information[25], the third (IC-a_0) is the incentive compatibility constraint that ensures that workers with assets $\hat{a}_0$ do not deviate and search in a market designed for workers with assets a_0 and the last constraint is the borrowing constraint of workers. Relative to the optimal contract from section 2, the new constraints are (Euler) and (IC-a_0). This formulation of the optimal contract is equivalent to the one used in section 2 but it simplifies the analysis when the initial asset a_0 is private information to workers.

[25]As is standard, we solve the optimal contract in which savings are private information using the first-order approach and verifying ex-post that the savings and search decisions satisfy global optimality.

Truthful reporting with CARA I first show that workers voluntarily select different contracts given their initial assets, so that firms can impute a_0 even if they do not observe it directly. The easiest way to establish this result is to consider the case of CARA utility, that is $u(c) = -\exp(-\gamma c)/\gamma$, because in this case the equilibrium in which asset a_0 is publicly observable coincides with the equilibrium in which it is private information, even though workers receive very different contracts across the asset distribution.

To see why, notice that with CARA utility equation (9) implies that the search decision and the EE rate of employed workers do not depend on their existing asset so that we can write them as $w^{ee}(w_1), p(w_1)$. As a result, the free entry condition (FE) in problem (10) does not depend on assets a anymore. This implies that the IC constraint (Euler) can be removed from problem (10) because it will always be satisfied, and that the feasibility set for $\lambda_0, w_0, w_1(x)$ does not depend on the worker savings decision a.

We now show that the constraint (IC-a_0) is slack, which proves that workers report their initial asset a_0 to firms truthfully. Specifically, we show that if the constraint (IC-a_0) is slack for all asset levels $\hat{a}_0 \neq a_0$, then it must also be slack for a_0. The value that workers with asset $\hat{a}_0$ obtains from truthfully reporting their type is

$$U_0(\hat{a}_0) = \max_{\hat{a}\geq 0, \hat{\lambda}_0, \hat{w}_0, \hat{w}_1(x)} \quad \hat{\lambda}_0 V_0(\hat{a}_0, \hat{a}, \hat{w}_0, \hat{w}_1(x)) + (1-\hat{\lambda}_0) 2u(b + \hat{a}_0/2)$$

where $\hat{\lambda}_0, \hat{w}_0, \hat{w}_1(x)$ must satisfy the free entry condition (FE). Then, it follows that

$$U_0(\hat{a}_0) \geq \max_{\hat{a}\geq 0} \lambda_0 V_0(\hat{a}_0, \hat{a}, w_0, w_1(x)) + (1-\lambda_0) 2u(b + \hat{a}_0/2)$$

since the combination $\lambda_0, w_0, w_1(x)$ chosen by workers with asset a_0 is feasible for workers with asset $\hat{a}_0$ and does not depend on their savings decision $\hat{a}$. In words, workers do not deviate to other labor markets because they could get the exact same contract and corresponding job finding rate if they wanted and still be free to optimize their savings decision. This condition is exactly the constraint (IC-a_0) for a worker with initial asset a_0 so this constraint is also slack for asset level a_0. This implies that workers report their initial assets truthfully to firms when utility is CARA[26].

[26]In the version of the model with more than 2 periods, the assumption that utility is CARA is not sufficient to ensure that workers reveal their initial assets a_0 to firms truthfully. The reason is that assets influences how backloaded wages can be *after an EE transition* so workers have an incentive to misreport their initial assets a_0 and savings decision to increase their EE rate. Even in this case, however, a worker with relatively more assets a_0 would select different contracts than a worker with relatively less assets a_0 because they can smooth their consumption better and thus seek contracts that provide less insurance but pay more on average. In general, the conditions for workers to report their initial assets a_0 truthfully are that the term $\mathcal{W}_t = 0$ in the Euler equation (7).

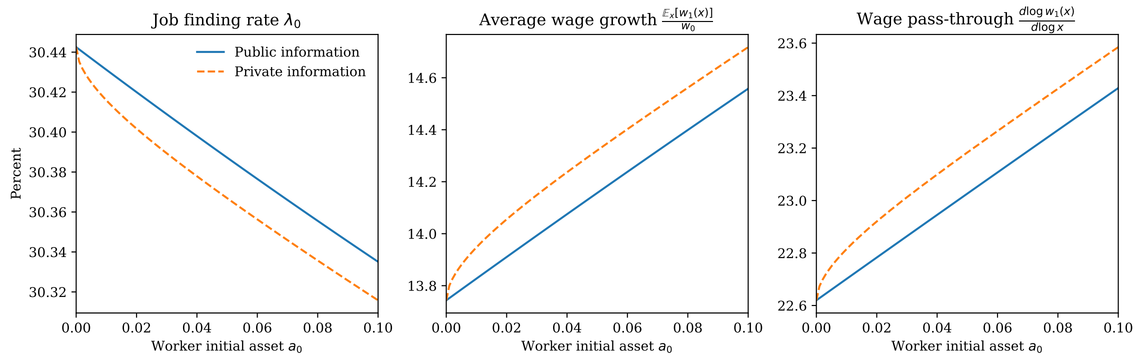
Note that in problem (10), workers with high initial assets a_0 receive wages that are more backloaded (higher $\mathbb{E}_x[w_1(x)]/w_0$) than workers with low initial assets. This means that wealthy workers are happy to receive wages that are more backloaded. The reason is that firms can offer higher average wages to wealthy workers when these wages are more backloaded because the EE rate is lower. Wealthy workers benefit from these higher average wages and smooth their consumption themselves using their existing assets. Meanwhile, poorer workers do not pretend to be wealthier to receive higher average wages because they could not smooth their consumption much if if their wages were very backloaded.

Separating equilibrium with CRRA Finally, I show that even if workers do not wish to report their initial assets truthfully, firms can modify their optimal contract slightly to induce truthful reporting by workers. I derive this result by computing the separating equilibrium with adverse selection when utility is CRRA, that is $u(c) = c^{1-\gamma}/(1-\gamma)$.

Consider first why workers benefit from misreporting their type using the optimality condition (9) and the free entry condition (FE). With CRRA utility, the search decision of workers depends on their assets because of standard wealth effects. Specifically, equation (9) shows that the search decision w^{ee} increases in the asset of workers a. As a result, the EE rate $p(w_1, a)$ is a decreasing function of assets. The free entry condition thus implies that a worker who decides to save more can receive a combination of higher wage $w_0, w_1(x)$ and higher job finding rate λ_0 because the retention rate, and thus firms' expected profits, are higher. Now consider how this influences the IC constraint (IC-a_0). A worker with initial asset a_0 is better off pretending to be a wealthier worker with $\tilde{a}_0 > a_0$ who is going to choose high savings a in order to receive this combination of higher wage $w_0, w_1(x)$ and higher job finding rate λ_0. Such a deviation is costly too because it means that the consumption of the worker will be more backloaded but overall the worker benefits from overstating his initial assets slightly[27].

[27] The gains from such deviations appear to be very small in this 2-period model: a worker with 0 initial assets who deviates increases his value as much as if he was not deviating but getting 10^{-7} more assets. Thus, another argument for why the model is robust to these deviations is that the benefits are so small that agents do not bother with them.

Figure 6: Separating equilibrium with hidden initial assets a_0



Note: the figure compared the equilibrium in which the worker initial asset a_0 is public information and the separating equilibrium in which it is private information but workers reveal their assets truthfully to firms. This exercise is done in a 2-period model where workers are employed in period 0 and can switch jobs in period 1. The left panel reports the job finding rate from unemployment at $t = 0$, the middle panel shows the average degree of wage backloading, and the right panel shows the pass-through of productivity shocks to wages at $t = 1$. The calibration is CRRA utility with $\gamma = 2$, $\delta = 0.1$, $b = 0.7$, $k = 1$, and $\kappa = 1$. The productivity distribution at $t = 1$ is discretized with 2 points $0.9, 1.1$. Taken together, these results show that firms induce truthtelling by workers by exposing wealthy workers to more risk relative to the model in which the worker initial asset a_0 is public information.

Consider now how firms modify optimal contracts to induce workers to truthfully reveal their initial assets a_0, which is shown in figure 6. The blue line shows the equilibrium in which the worker initial asset a_0 is public information whereas the dotted orange line shows the equilibrium in which the asset is private information but workers end up revealing their type truthfully to firms. The figure shows that the job finding rate falls faster with initial assets a_0 in the equilibrium with private information than with public information (left panel), workers with higher assets a_0 receive wages that are more backloaded (middle panel) and with a higher pass-through (right panel). Thus, firms induce truthtelling by workers by making contracts riskier for wealthy workers. The single crossing property that leads to separation in equilibrium is that making the contract more risky is especially costly to poor workers with little assets a_0.

Taken together, this section shows that even if firms cannot infer perfectly the worker's assets from their observed characteristics, such as occupation, the equilibrium would not unravel since a minor modification of the optimal contract would induce workers to sort themselves into contracts with different degrees of risk.

6 Conclusion

This paper builds a new model with optimal wage contracts, assets and search frictions. The insurance that workers receive outside the firm, through financial markets, significantly crowds out the insurance that they receive inside the firm, through optimal wage contracts. As a result, wealthy workers receive wages that are more backloaded are more volatile relative to poor workers, but they are also matched with more productive firms and receive higher average wages. This model has novel implications for policies that relax borrowing constraints because these policies enable poor workers to receive wage contracts that are similar to those received by wealthy workers. The model built in this paper is the first to combine optimal wage contracts with realistic financial markets. As such, it can be used as a foundation for future work studying how business cycles influence the earnings and consumption of different workers.

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Appendix

Contents

A Model appendix

A.1 Proof of proposition 1

A.1.1 Part 1

Consider any allocation that solves the optimal contract with a path for assets $a_{t+1}(x^t)$, where x^t denote the history of productivity since the beginning of the match. We can construct a new path wages $\tilde{w}_t(x^t)$ and transfers $\tilde{\tau}_t^{ee}(x^t)$, $\tilde{\tau}_t^{eu}(x^t)$ such that the allocations $c_t(x^t)$, $p_t(x^t)$ remain the same and solve the optimal contract while the path for assets satisfies $\tilde{a}_{t+1}(x^t) = 0$.

Let the new paths of transfers be $\tilde{\tau}_t^{ee}(x^t) = \tau_t^{ee}(x^t) - a_t(x^{t-1})$ and $\tilde{\tau}_t^{eu}(x^t) = \tau_t^{eu}(x^t) + a_t(x^{t-1})$. The new path of wages is $\tilde{w}_t(x^t) = (1+r)a_t(x^{t-1}) + w_t(x^t) - a_{t+1}(x^t)$ for all periods except the first, and $\tilde{w}_t = w_t - a_{t+1}$ for the first period. This wage ensures that the path of consumption $c_t(s^t)$ can remain the same under the new contract.

First, we show that the worker value remains constant after any history with this new contract. The worker value satisfies

$$\begin{aligned} \tilde{V}_t(x^t) = &\ \delta U(\tilde{\tau}_t^{eu}(x^t)) + (1-\delta)\left[\tilde{W}_t(x^t) + S(\tilde{W}_t(x^t), -\tilde{\tau}_t^{ee}(x^t))\right] \\ = &\ \delta U(a_t(x^{t-1}) + \tau_t^{eu}(x^t)) + (1-\delta)\left[\tilde{W}_t(x^t) + S(\tilde{W}_t(x^t), a_t(x^{t-1}) - \tau_t^{ee}(x^t))\right] \end{aligned}$$

with $\tilde{W}_t(x^t) = u(c_t(x^t)) + \beta \mathbb{E}_{x_{t+1}}\left[\tilde{V}_{t+1}(x^{t+1}) | x^t\right]$. This shows that $\tilde{V}_t(x^t) = V_t(x^t)$ and $\tilde{W}_t(x^t) = W_t(x^t)$ after any x^t.

Then, we show that this new contract achieves the same *initial* value to firms (note that the firm value will differ after some histories). The firm value after history x^t under the original contract satisfies

$$
\begin{aligned}
\Pi_t(x^t) = \quad & (1-\delta)\left(1-p(W_t(x^t),a_t(x^{t-1})-\tau_t^{ee}(x^t))\right)\left(x_t - w_t(x^t) + \tfrac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right) \\
& -\delta(1+r)\tau_t^{eu}(x^t) + (1-\delta)p(W_t(x^t),a_t(x^{t-1})-\tau_t^{ee}(x^t))(1+r)\tau_t^{ee}(x^t) \\
= \quad & (1-\delta)\left(1-p(W_t(x^t),-\tilde{\tau}_t^{ee}(x^t))\right)\left(x_t - \tilde{w}_t(x^t) + (1+r)a_t(x^{t-1}) - a_{t+1}(x^t) + \tfrac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right) \\
& -\delta(1+r)\left(\tilde{\tau}_t^{eu}(x^t) - a_t(x^{t-1})\right) + (1-\delta)p(W_t(x^t),-\tilde{\tau}_t^{ee}(x^t))(1+r)\left(\tilde{\tau}_t^{ee}(x^t) + a_t(x^{t-1})\right)
\end{aligned}
$$

Now define $\tilde{\Pi}_t(x^t) = \Pi_t(x^t) - (1+r)a_t(x^{t-1})$ for all x^t. Use $\tilde{W}_t(x^t) = W_t(x^t)$ in the previous equation to get

$$
\begin{aligned}
\tilde{\Pi}_t(x^t) = \quad & \Pi_t(x^t) - (1+r)a_t(x^{t-1}) \\
= \quad & (1-\delta)\left(1-p(\tilde{W}_t(x^t),-\tilde{\tau}_t^{ee}(x^t))\right)\left(x_t - \tilde{w}_t(x^t) + \tfrac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right) \\
& -\delta(1+r)\tilde{\tau}_t^{eu}(x^t) + (1-\delta)p(\tilde{W}_t(x^t),-\tilde{\tau}_t^{ee}(x^t))(1+r)\tilde{\tau}_t^{ee}(x^t)
\end{aligned}
$$

which shows that $\tilde{\Pi}_t(x^t)$ is precisely the firm value under the alternative contract.

Now plug this in the problem of new entrants to verify that the firm value is the same in the first period

$$
\begin{aligned}
\tilde{\Pi}_0(v,a_t,x_0) = \quad & x_0 - \tilde{w}_t + \tfrac{\mathbb{E}_{x_{t+1}}\left[\tilde{\Pi}_{t+1}(x^{t+1})|x_0\right]}{1+r} \\
= \quad & x_0 - w_t + a_{t+1} - a_{t+1} + \tfrac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x_0\right]}{1+r} \\
= \quad & x_0 - w_t + \tfrac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x_0\right]}{1+r} \\
= \quad & \Pi_0(v,a_t,x_0)
\end{aligned}
$$

Therefore, the new contract with $a_{t+1}(x^t) = 0$ after any x^t achieves the same initial firm value. The allocations satisfies the constraints on the optimal contract every period, including the borrowing constraint, so the new contract is also a solution to the optimal contract. Finally, the EE separation rate $p_t(x^t)$ remains the same since

$$
p(\tilde{W}_t(x^t),-\tilde{\tau}_t^{ee}(x^t)) = p(W_t(x^t),a_t(x^{t-1})-\tau_t^{ee}(x^t))
$$

A.1.2 Part 2

I prove part 2 under the assumption that productivity is constant within matches so I will denote the wage as w_t, where t denotes time. Denote the allocation achieved with transfers as c_t^*, p_t^* and the corresponding transfers as $(\tau_t^{ee})^*, (\tau_t^{eu})^*$. We assume that $a_{t+1}^* = 0$, which is without loss of generality from part 1.

We now construct a new contract consisting of paths for wages, assets and transfers $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ such that

a) $\tau_t^{ee} = 0$ for all t,

b) the worker consumption c_t and values V_t, W_t are the same for all t,

c) the worker EE separation rate $p_t \equiv p(W_t, a_t - \tau_t^{ee})$ is the same for all t,

d) the initial firm value $\Pi_0(v,a,x_0)$ is the same.

This new contract delivers the same value to firms and satisfies all the constraints of the relaxed problem in which transfers τ_t^{ee} can be implemented (note that there is no borrowing constraint in part 2). Since this contract also satisfies the constraint $\tau_t^{ee} = 0$, it must solve the optimal contract where transfers τ_t^{ee} cannot be implemented. Thus, it would show that we can implement the same allocation with a contract that satisfies $\tau_t^{ee} = 0$ provided that workers can use assets.

Construct the new contract $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ using

$$w_t = w_t^* - (1+r)a_t + a_{t+1}$$

$$a_{t+1} = (\tau_{t+1}^{ee})^*$$

$$\tau_t^{ee} = 0$$

$$\tau_t^{eu} = (\tau_t^{eu})^* - a_t$$

except for the first period where $w_t = w_t^* + a_{t+1}$. The consumption of the worker implied by this path is unchanged since

$$c_t = (1+r)a_t + w_t - a_{t+1} = w_t^* = c_t^*$$

for all periods except the first, and

$$c_t = (1+r)a_0 + w_t - \tilde{a}_{t+1} = (1+r)a_0 + w_t^* = c_t^*$$

for the first period.

We now show that the worker value remains constant after any history with this new contract. The worker value satisfies

$$\begin{aligned} V_t = & \quad \delta U(a_t + \tau_t^{eu}) + (1-\delta)\left[W_t + S(W_t, a_t)\right] \\ = & \quad \delta U((\tau_t^{eu})^*) + (1-\delta)\left[W_t + S(W_t, (\tau_t^{ee})^*)\right] \end{aligned}$$

with $W_t = u(c_t^*) + \beta V_{t+1}$. This shows that $V_t = V_t^*$ and $W_t = W_t^*$ for all t. It also follows that the EE rate p_t remains the same since

$$p_t \equiv p(W_t, a_t) = p(W_t^*, \tau_t^{ee})$$

Finally, consider the firm value with these paths, denoted Π_t. It satisfies

$$\begin{aligned} \Pi_t = & \quad (1-\delta)\left(1 - p(W_t, a_t)\right)\left(x_0 - w_t + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)\tau_t^{eu} \\ = & \quad (1-\delta)\left(1 - p(W_t^*, (\tau_t^{ee})^*)\right)\left(x_0 - w_t^* + (1+r)a_t - a_{t+1} + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)(\tau_t^{eu})^* + \delta(1+r)a_t \\ = & \quad (1-\delta)\left(1 - p(W_t^*, (\tau_t^{ee})^*)\right)\left(x_0 - w_t^* - a_{t+1} + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)(\tau_t^{eu})^* \\ & \quad +(1+r)a_t + (1-\delta)p(W_t^*, (\tau_t^{ee})^*)(1+r)(\tau_t^{ee})^* \end{aligned}$$

Now guess and verify that $\Pi_t = \Pi_t^* + (1+r)a_t$. Going back to the first period, we get

$$\begin{aligned} \Pi_0(v, a_t, x_0) = & \quad x_0 - w_t + \tfrac{\Pi_{t+1}}{1+r} \\ = & \quad x_0 - w_t^* - a_{t+1} + a_{t+1} + \tfrac{\Pi_{t+1}^*}{1+r} \\ = & \quad x_0 - w_t^* + \tfrac{\Pi_{t+1}^*}{1+r} \\ = & \quad \Pi_0^*(v, a_t, x_0) \end{aligned}$$

and therefore the initial value of the firm is the same with this new contract.

A.1.3 Part 3

The proof is similar to part 2 except that the new contract satisfies $\tau_t^{eu} = 0$ for all t. The new contract $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ satisfies

$$w_t = w_t^* - (1+r)a_t + a_{t+1}$$

$$a_{t+1} = (\tau_{t+1}^{eu})^*$$

$$\tau_t^{ee} = (\tau_t^{ee})^* + a_t$$

$$\tau_t^{eu} = 0$$

except for the first period where $w_t = w_t^* + a_{t+1}$. From there, the proof is identical to part 2.

A.2 Consumption growth condition (5)

Consider the optimal contract with $\tau_t^{ee} = \tau_t^{eu} = 0$ and denote the Lagrange multipliers on the constraints as $\eta_t, \lambda_t, \mu_t, \zeta_t$. The optimality conditions are

$$\begin{aligned}
w_t: &\quad (1-\delta)(1-p(W_t,a_t)) = \mu_t \\
c_t: &\quad \lambda_t u'(c_t) = \mu_t \\
V(x_{t+1}): &\quad (1-\delta)(1-p(W_t,a_t))(1+r)^{-1}\Pi_V(s_{t+1}) + \beta\lambda_t = 0 \\
W_t: &\quad -p_W(W_t,a_t)\left(x_t - w_t + \tfrac{\mathbb{E}_{x_{t+1}}[\Pi(s_{t+1})|x_0,x_t]}{1+r}\right) + \eta_t(1-p(W_t,a_t)) - \tfrac{\lambda_t}{1-\delta} = 0 \\
a_{t+1}: &\quad (1-\delta)(1-p(W_t,a_t))(1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi_a(s_{t+1})|x_0,x_t\right] = \mu_t - \zeta_t
\end{aligned}$$

and the envelope conditions are

$$\begin{aligned}
V_t: &\quad \Pi_V(s_t) = -\eta_t \\
a_t: &\quad \Pi_a(s_t) = -(1-\delta)p_a(W_t,a_t)\left(x_t - w_t + (1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]\right) \\
&\qquad + \eta_t\left(\delta U'(a_t) + (1-\delta)S_a(W_t,a_t)\right) + \mu_t(1+r)
\end{aligned}$$

Consider the optimality conditions with respect to $V(x_{t+1})$ and W_t in the optimal contract, and the envelope condition with respect to V_t to get

$$\beta^{-1}(1+r)^{-1}\eta_{t+1} - \eta_t = -\frac{p_W(W_t,a_t)}{1-p(W_t,a_t)}\left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]}{1+r}\right)$$

Now combine the first-order conditions for c_t, w_t and $V(x_{t+1})$ and the envelope condition for V_t to get

$$\eta_{t+1} = \frac{\beta(1+r)}{u'(c_t)} \tag{A.1}$$

and therefore

$$\frac{1}{u'(c_t)} - \frac{\beta(1+r)}{u'(c_{t-1})} = -\frac{p_W(W_t,a_t)}{1-p(W_t,a_t)}\left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]}{1+r}\right)$$

which is equation (5).

A.3 Proof of proposition 2

First, combine the first-order condition for a_{t+1} with the envelope condition for a_t to get

$$\begin{aligned} 1+r \geq\ & \mathbb{E}_{x_{t+1}}\left[\eta_{t+1}\left(\delta U'(a_{t+1}) + (1-\delta)S_a(W_{t+1},a_{t+1})\right) + (1-\delta)(1-p(W_{t+1},a_{t+1}))(1+r)|x_t\right] \\ & -\mathbb{E}_{x_{t+1}}\left[(1-\delta)p_a(W_{t+1},a_{t+1})\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned} \tag{A.2}$$

with equality if the borrowing constraint does not bind.

Next, combine the first-order condition for c_t and the envelope condition of the unemployed workers to get

$$U'(a_t) = (1+r)u'(c_t^u) \tag{A.3}$$

where c_t^u denotes the consumption of the unemployed worker.

Finally, we need to derive an expression for $S_a(W_{t+1}, a_{t+1})$. From the envelope condition on the search problem of workers (2), we get

$$S_a(W_t,a_t) \equiv \kappa\left(v(W_t,a_t) - W_t\right)\partial_a\lambda_w(v(W_t,a_t),a_t)$$

We now derive an expression for $\partial_a\lambda_w(v(W_t,a_t),a_t)$ from the free entry condition of firms. Combining the first-order conditions and envelope conditions in the problem of new entrants gives

$$\partial_v\Pi_0(v,a_t) = -\frac{1}{u'(c_t)}$$

$$\partial_a\Pi_0(v,a_t) = 1+r$$

where c_t represents the consumption of the worker at the new job. Now consider the free entry condition

$$\lambda_f(v,a) = \frac{k}{\Pi_0(v,a)}$$

Differentiating this expression with respect to v and a gives

$$\partial_a\lambda_f(v,a) = -\lambda_f(v,a)\frac{1+r}{\Pi_0(v,a)}$$

$$\partial_v\lambda_f(v,a) = \lambda_f(v,a)\frac{1}{\Pi_0(v,a)u'(c_t)}$$

Taking the ratio gives

$$\partial_a\lambda_f(v,a) = -\partial_v\lambda_f(v,a)(1+r)u'(c_t)$$

With a constant returns to scale matching function, we can express the job finding rate as $\lambda_w(v,a) = f(\lambda_f(v,a'))$. Therefore,

$$\partial_a\lambda_w(v,a) = -\partial_v\lambda_w(v,a)(1+r)u'(c_t) \tag{A.4}$$

We can use this expression to rewrite $S_a(W_t,a_t)$ as

$$S_a(W_t,a_t) = -(1+r)u'(c_t^{ee})\kappa\left(v(W_t,a_t) - W_t\right)\partial_v\lambda_w(v(W_t,a_t),a_t)$$

where c_t^{EE} is the consumption of the worker during the first period after an EE separation. We can further simplify this term using the first-order condition of the search problem

$$\lambda_w(v_t, a_t) + \partial_v \lambda_w(v_t, a_t)\,(v_t - W_t) = 0$$

and get

$$S_a(W_t, a_t) = (1+r)u'(c_t^{ee})\kappa\lambda_w(v(W_t, a_t), a_t) = (1+r)u'(c_t^{ee})p(W_t, a_t) \tag{A.5}$$

Combine equations (A.2), (A.1), (A.3), and (A.5), to get

$$\begin{aligned} u'(c_t) \geq\ & \mathbb{E}_{x_{t+1}}\left[\beta(1+r)\left(\delta u'(c_{t+1}^u) + (1-\delta)p(W_{t+1}, a_{t+1})u'(c_{t+1}^{ee})\right) + (1-\delta)(1-p(W_{t+1}, a_{t+1}))u'(c_t)|x_t\right] \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\frac{p_a(W_{t+1}, a_{t+1})}{1+r}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

For the final step, rewrite the consumption growth condition (5) evaluated at $t+1$ as

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) - u'(c_t)u'(c_{t+1})\frac{p_W(W_{t+1}, a_t)}{1-p(W_{t+1}, a_t)}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}\left[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}\right]}{1+r}\right)$$

We can replace $u'(c_t)$ on the right-hand side by this expression and get

$$\begin{aligned} u'(c_t) \geq\ & \beta(1+r)\left(\delta u'(c_{t+1}^u) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p(W_{t+1}, a_{t+1})u'(c_{t+1}^{ee}) + (1-p(W_{t+1}, a_{t+1}))u'(c_{t+1})|x_t\right]\right) \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\left(u'(c_{t+1})p_W(W_{t+1}, a_t) + \frac{p_a(W_{t+1}, a_{t+1})}{1+r}\right)\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

which is equation (7).

A.4 Optimality conditions for unemployed workers

The optimality condition with respect to v_{t+1} leads to the standard optimality condition in directed search models

$$\lambda_w(v_{t+1}, a_{t+1}) = -\partial_v \lambda_w(v_{t+1}, a_{t+1})\,[v_{t+1} - U(a_{t+1})]$$

It states that workers equate the benefits of search in markets with higher values v_{t+1} to the cost in terms of decreased match probability.

Combining the optimality conditions with respect to c_t and a_{t+1} and the envelope condition gives

$$u'(c_t) \geq \beta\partial_a\lambda_w(v_{t+1}, a_{t+1})\,[v_{t+1} - U(a_{t+1})] + \beta(1-\lambda_w(v_{t+1}, a_{t+1}))(1+r)u'(c_{t+1})$$

Finally, we can use equation (A.4) and the optimality condition for v_{t+1} to get

$$u'(c_t) \geq \beta(1+r)\left[\lambda_w(v_{t+1}, a_{t+1})u'(c_{t+1}^{ue}) + (1-\lambda_w(v_{t+1}, a_{t+1}))u'(c_{t+1})\right]$$

which is a standard Euler equation.

A.5 Optimal contract with hidden savings

I solve the optimal contract under the assumption that the savings decision of workers is private information using the first-order approach (Werning, 2002, Abraham and Pavoni, 2008), which means that the Euler equation is now an additional constraint in problem (3). The challenge here is that the problem must be solved with high accuracy to compare the solution to the problem in which assets are public information because the two solutions are so similar. At the same time, solving the problem with hidden savings requires an additional state variable (the marginal utility of consumption yesterday) and an additional choice variable (the marginal utility of consumption today). To keep to computations feasible, I first compute the equilibrium job finding rate $\lambda_w(v, a)$ and value of unemployment U from the problem where assets are public information, and I use those to solve for the optimal contract with hidden savings. I also focus on a simplified model where productivity is constant across firms and over time to reduce the state space.

I compute the difference between the paths of wages, consumption, assets and EE transition rate in the model with hidden savings and the model with observable savings. I find that the differences are respectively of the order of 10^{-5}, 10^{-5}, 10^{-3} and 10^{-5}. I also check ex-post that the job finding rate implied by the optimal contract computed under the assumption of hidden savings is similar to the one computed with observable savings (difference of 10^{-3}). Therefore, these results show that the solutions to the model with hidden savings or observable savings are virtually identical. This result is not surprising given that Shimer and Werning (2008) also find that hidden savings made no difference using a similar model with hidden search in the context of unemployment insurance.

B Data appendix

I combine information on wages, worker mobility and firm productivity from French matched employer-employee data with information on liquid assets from the Household Finance and Consumption Survey and information on unemployment benefits from the OECD.

B.1 Matched employer-employee data

I use administrative data provided by the CASD in France between 2008 and 2019. I use this data for information about wages, worker mobility and firm productivity.

My analysis relies on two main files:

a) the panel version of the “DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) “FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms (“Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level (“UL"), and several legal units can belong to the same firm. Since I want to measure EE separations across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the “Entreprise profilée" (“EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years and for firms with at least 3 employees (in the panel or not). I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.6}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying EE separations is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job separation. An EE separation occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as EE separations if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of EE separations with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make an EE separation experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job separations with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

When a worker separates from their previous jobs and does not make an EE separation, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. separation from private sector to public sector), I do not count it either as an EE separation nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.

B.2 Liquid assets

I compute liquid assets using the 2014 wave of the Household Finance and Consumption Survey for France. I keep households whose head is between 25 and 55 years old and who is either employed or unemployed. Liquid wealth is divided by net income, defined as labor income plus unemployment benefits minus income and social security taxes. I then compute the median liquid asset to income ratio both in the data and in the model. It is well known that the median asset to income ratio is less sensitive to the right tail of the wealth distribution, which precautionary savings models are not able to match.

B.3 Unemployment value

I use data on the net replacement rates in unemployment published by the OECD for France. The data shows that the replacement ratio is 0.68 for workers unemployed for 24 months or less, and 0.24 later on. To compute the average replacement ratio in the data, I compute the share of unemployed workers who have been unemployed for less than 24 months in my model and I use this share to average between 0.68 and 0.24. I find that the replacement ratio is 0.62 in the data. I then set the value of home production b to match that estimate.