Insurance Inside and Outside the Firm

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Abstract

This paper presents a new model with optimal wage contracts, assets, and search frictions. Contracts are subject to hidden search and limited worker commitment, and workers can trade risk-free bonds subject to borrowing constraints. Assets play two roles in wage contracts. First, they allow firms to backload wages more and thus to retain workers more easily. Second, they are used to insure workers against unemployment risk, which is easier when wages are frontloaded. The optimal contract balances these forces and features tenure profiles for wages and worker mobility across jobs that are consistent with evidence from matched employer-employee data from France. Relative to a model without assets, the pass-through of firm-level productivity shocks to wages is three times larger with assets but the pass-through to consumption is two times smaller. Thus, when workers have access to insurance outside the firm in the form of risk-free bonds, firms provide less insurance to workers against productivity shocks but workers receive more insurance overall.

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1 Introduction

How do firms and workers share risk when profits fluctuate? Do firms absorb productivity shocks into their profits or do they pass them on to workers through their wages? An enduring idea in economics is that firms provide some form of insurance to workers through wage contracts (Knight, 1921, Baily, 1974, Azariadis, 1975). Estimates of the pass-through of productivity shocks to wages can be used to assess whether firms actually insure workers against these shocks or pass them through to wages. Recent evidence shows that the pass-through is indeed small but positive (Guiso, Pistaferri and Schivardi, 2005), suggesting that firms provide significant but incomplete insurance to workers.

The existing literature has studied this risk-sharing problem between firms and workers using models with optimal wage contracts (e.g. Thomas and Worrall, 1988, Beaudry and DiNardo, 1991, Balke and Lamadon, 2022). These models, however, make the stark assumption that workers have no access to financial markets so that firms are the only source of insurance in the economy. This assumption is at odds with evidence that households hold liquid assets and that their marginal propensity to consume out of income shocks is less than 100%. There is also an influential literature studying how workers self-insure through financial markets (e.g. Bewley, 1977, İmrohoroğlu, 1989, Huggett, 1993, Aiyagari, 1994), but these models assume that wage contracts are exogenous and thus that firms play no role in insuring workers against shocks. As a result, we know very little about the interaction between the insurance that workers receive *inside the firm*, through wage contracts, and the insurance that workers receive *outside the firm*, through financial markets.

This paper builds a new model with search frictions, optimal wage contracts and assets in general equilibrium. The model brings together the dynamic wage contracting model of Menzio and Shi (2010) with precautionary savings models in the tradition of Bewley (1977). I find that assets influence wage contracts in two ways. First, assets allow firms to backload wages more and thus to retain workers more easily. Second, assets are used to insure workers against unemployment risk, which is easier when wages are frontloaded. The optimal contract balances these forces and features tenure profiles for wages and worker mobility across jobs that are consistent with evidence from matched employer-employee data from France. The equilibrium features rich interactions between labor market mobility and the risk that workers face from unemployment and on the job. I use the quantitative model to revisit the pass-through of productivity shocks to wages and consumption, and measure how much insurance workers receive inside the firm as opposed to outside the firm against these shocks. I assume in most of the analysis that assets are public information but later show that optimal deviations are small when assets are private information.

The model features risk-averse workers and dynamic wage contracts with directed search. Workers transition between employment and unemployment, and can switch jobs. Contracts are subject to limited commitment on the side of workers, and there is imperfect information about the worker search decision. Specifically, firms cannot observe the search decision of workers and workers cannot commit to make any payment to previous employers after an employer-to-employer (EE) transition. The hidden search assumption leads to moral hazard, and how firms respond to it is constrained by the limited commitment of workers. Workers can trade non-contingent assets subject to a borrowing constraint. In the baseline, these trades are public information so firms and workers can contract upon them and firms also know the asset of the workers they match with. Workers and firms face two sources of risk. First, firms face firm-level productivity shocks and choose as part of the contract how much to optimally pass them on to workers. Second, workers face exogenous unemployment shocks. Critically, I assume that unemployment shocks are not insurable in the sense that firms are not allowed to make severance payments to workers. This restriction plays a critical role for the optimal contract because it implies that non-contingent assets brings insurance benefits to workers beyond what firms can achieve.

The optimal contract implies an extreme degree of wage backloading when workers do not face such uninsurable unemployment risk. Specifically, when unemployment risk is nonexistent or insurable by firms, the optimal contract implies a low or even negative wage during the first period of employment followed by much higher wages later on. As a result, workers use all their existing assets initially to smooth consumption. To understand why this is optimal, it is useful to remember the intuitions behind the standard wage backloading result in contracting models without assets (e.g. Burdett and Coles, 2003, Shi, 2009). Firms typically want to retain their workers, but cannot control their search decision. As a result, they find it optimal to backload wages to make workers search for jobs with higher wages and lower job finding rates. However, to backload wages is to backload consumption in models without assets, which makes contracts relatively unattractive to risk-averse workers. Indeed, if one firm adopted a strategy of extreme wage backloading, it would have to offer much higher average wages to make its offer attractive relative to an offer that offers more stable wages and, hence, more stable consumption. By contrast, when workers are allowed to trade assets firms can backload wages more without backloading consumption by making workers consume their assets initially. As a result, wages are significantly more backloaded when workers have positive asset holdings.

There is a important connection between this result on extreme wage backloading and the literature on optimal unemployment insurance. This literature (e.g. Shavell and Weiss, 1979, Hopenhayn and Nicolini, 1997) studies an optimal insurance contract that is very similar to the optimal wage contract that I study. From this perspective, it might be surprising that assets influence my wage contract so much even when they are public information, whereas it is well known that they have no effect on the optimal allocation in the unemployment insurance literature (hence the focus of this literature on hidden savings). I show that this difference arises because workers have limited commitment in my model. Instead, in the unemployment insurance literature the government can enforce transfers from workers after unemployment-to-employment (UE) transitions using taxes, which is equivalent to assuming that workers have commitment. In fact, I show that when workers face no unemployment risk, no borrowing constraint and when productivity is constant within matches, the allocation is exactly identical in my model with limited worker commitment and assets, and in a model with worker commitment. Even though the allocation is identical, the implementations are very different. With limited commitment, the contract is implemented with a negative wage in the first period and negative asset holdings. With commitment, it is implemented with transfers from workers to previous employers after EE transitions. In this sense, assets act as a substitute for worker commitment in the optimal contract.

When workers face uninsurable unemployment risk, as in my quantitative model, assets are also a vehicle for precautionary savings. In particular, workers who become unemployed receive low unemployment benefits. If those workers have no asset, they will experience a large drop in consumption. Thus, backloading wages too much and making workers consume their savings at the beginning of a job makes contracts very unattractive to risk-averse workers who anticipate that their consumption will fall significantly if an unemployment shock occurs. The optimal contract thus implies a moderate degree of wage backloading that depends on the competition for workers and the uninsurable risk that workers face. In the extreme case where workers cannot switch jobs, the optimal contract even implies an extreme degree of wage frontloading. Firms pay workers a large wage in the first period of employment, which allows workers to accumulate enough savings that their marginal propensity to consume out of unemployment shock is zero. This means that without contracting frictions related to worker mobility across jobs, optimal contracts undo the market incompleteness at the core of precautionary savings models.

I discipline the model with French administrative data to quantify how firms balance the motive to backload wages to retain workers, and the motive to frontload wages to help workers self-insure. I estimate model parameters using moments that are informative about the risk that workers face from unemployment, and the extend to which firms are competing to retain their workers. Specifically, I target worker transitions across employment and unemployment (UE, EE and EU rates), the tenure profile of wages and EE transitions, the standard deviation and persistence of firm productivity shocks, and an estimate of the marginal propensity to consume out of unemployment shock. In the context of France, I find that the combination of low worker mobility across jobs and moderate unemployment risk leads to a modest degree of wage backloading.

I perform three comparatives static exercises to quantify how the competition for workers and unemployment risk influence wage contracts. First, I increase the EE transition rate by half, which enhances the competition for worker and makes wages more backloaded. As a result, workers accumulate less assets within a job but are also more likely to switch jobs, which helps them accumulate assets. Overall, I find that asset holdings increase by 20% on average, so workers are better insured against unemployment, but workers with little job market experience hold less assets and suffer larger drops in consumption after unemployment shocks. Second, I reduce unemployment benefits by a third, which increases unemployment risk and makes the job ladder longer as unemployed workers search for jobs with lower wages. I find that as a result of the policy change, the consumption of workers becomes more volatile not just after an unemployment shock but also while they are employed. Third, I quantify the gains from optimal wage contracts by measuring how much profit a single firm would generate if it were to post fixed-wage contracts in which wages are constant over time and across states, similar to Burdett and Mortensen (1998). I find that relative to optimal contracts, posting fixed-wage contracts is equivalent to reducing productivity permanently by 7%. Thus, optimal contracts yield significant benefits to firms relative to simple contracts.

Finally, I use the quantitative model to revisit the pass-through of persistent productivity shocks to the consumption and wage of workers. The model captures a trade-off between retaining workers when they are most productive, and insuring them against productivity shocks. Risk-neutral firms would like to insure risk-averse workers against shocks because it allows them to attract workers despite offering lower average wages. The best way to insure workers is to keep their consumption independent of productivity. However, with such a strategy workers would leave at a constant rate. The firm can increase its profits by raising the consumption of workers when productivity is high to reduce the quit rate, and lowering consumption when productivity is low to increase it. With this strategy firms would retain workers precisely when they generate the most profits. Therefore, firms balance the benefits of varying consumption with productivity to optimize worker retention against the benefits of providing insurance to workers against shocks so as to design contracts that both maximize profits and are attractive to workers.

What path of wages implements the optimal pass-through of shocks to consumption? Consider for instance a positive shock to productivity. I find that firms respond by cutting the wage of workers on impact, and increasing it significantly later on. Meanwhile, consumption increases smoothly over time as workers initially use their assets in anticipation of higher future income. Eventually, the wage of workers increases so much that workers accumulate more assets. Why is it optimal for firms to cut wages and deplete the worker assets on impact after a positive productivity shock? When productivity increases, the profits of firms rise. As a result, the worker retention motive described before becomes stronger than the precautionary savings motive, and firms choose to backload wages more. They backload wages so much that wages actually fall on impact. Quantitatively, I find that the pass-through to wages is 3 times higher relative to a model in which workers have no access to financial markets, but that the pass-through to consumption is about 2 times lower. Therefore, when workers have access to insurance outside the firm they receive less insurance from firms but more insurance overall.

Throughout, I have assumed that assets are public information. This implies that firms know how much assets workers have when they first match, and that the saving decision of workers is contractible. It is natural to wonder whether this assumption, which is somewhat unrealistic, has important implications for the optimal contract. I show that with CARA utility, the allocation is identical when assets are private or public information. This result implies that we can reinterpret the model as one where firms design wage contracts that workers select depending on their assets, and where workers choose how much to consume and save independently of firms. With CRRA utility however, the optimal contract designed under the assumption that assets are public information is no longer incentive compatible because of wealth effects on search. These deviations however appear to be quantitatively small, suggesting that the optimal contract would not be very different if they were taking into account these deviations. Those results are closely related, thought not identical, to those of Werning (2002) and Abraham and Pavoni (2008) on optimal unemployment insurance, and of Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023) on fixed-wage contracts with assets and directed search.

In this paper I focus on non-contingent assets, as opposed to more general securities, because in the data most assets held by households, such as cash, are non-contingent. Nevertheless, it is worth comparing my results to those obtained by Stevens (2004) in a similar model where workers have access to complete financial markets. With complete markets, the optimal contract is implemented by an upfront fee paid by workers so it also features an extreme degree of wage backloading. However, an important difference is that with complete markets the optimal contract no longer suffers from moral hazard. By contrast, with non-contingent assets, the optimal contract is still subject to moral hazard because firms want to insure workers against the risk of not finding another job[1]. Besides, the model with complete markets makes predictions that are at odds with the data. In particular, in my model the tenure profile of wages is consistent with the data precisely because workers face uninsurable unemployment risk, and there is limited pass-through of productivity shocks to wages, as in the data, only with incomplete markets.

This paper contributes to a recent literature bringing together labor market transitions and asset accumulation. An early example is Krusell, Mukoyama and Şahin (2010) who study precautionary savings in a DMP model where wages are set by Nash bargaining. Several articles have since introduced assets in search models with EE transitions (Lise, 2013, Chaumont and Shi, 2022, Alves, 2022, Kaas, Lalé and Nawid, 2023, Caratelli, 2024) but in these models wage contracts are subject to ad hoc restrictions. Specifically, wages are assumed to be constant during matches, or assumed to change only when workers receive an outside offer. Instead, my paper is the first to study the determinants of worker mobility and assets in a model with optimal wage contracts. I show that fixed-wage contracts are inefficient because they abstract from the firm's desire to retain workers and the worker's demand for precautionary savings. Quantitatively, I find the gains from optimal contracts relative to fixed-wage contracts to be quite large.

The paper starts in section 2 by presenting a new model with wage contracts and assets, which I characterize in section 3. Section 4 brings the model to data and quantifies the insurance that workers receive inside and outside the firm. Finally, section 5 revisits the assumption that assets are public information. Proofs are in the appendix.

2 A model with wage contracts and assets

I first present a new model with search frictions, dynamic wage contracts and assets. The model brings together the models of Menzio and Shi (2010) and Bewley (1977).

2.1 Environment

Time is discrete and runs forever.

[1]This is similar to what Shimer and Werning (2008) call the need to "insure against uncertain spell duration" in the unemployment insurance literature.

Agents A continuum of ex-ante homogeneous workers can be employed or unemployed. Workers receive wage w when employed, and home production b when unemployed. They have period utility $u(c)$ over consumption and discount the future at rate β.

Firm are owned by foreign diversified investors, so they are effectively risk-neutral with discount rate r. An active firm is one that is matched with a single worker. The output from that match is x_t with firm productivity x_t following the mean reverting process

$$x_t = (1-\rho)\overline{x} + \rho_x x_{t-1} + \sigma_x \nu_t$$

where ν_t are i.i.d. innovations with standard normal distribution, and ρ_x parameterizes the persistence of productivity. Firm fixed productivity $\overline{x}$ is drawn at the start of the match, independently across firms. This productivity stays constant over time and lasts for the length of the match.

Financial markets This is a small open economy with foreign interest rate r. Workers can save using risk-free bonds a_{t+1} subject to a borrowing constraint, so that

$$a_{t+1} \geq 0$$

Timing Each period, the sequence of events is as follows

a) Firm productivity shocks and exogenous separations into unemployment occur

b) Employed and unemployed workers (at the start of the period) search for jobs; firms post vacancies; new matches are formed and new contracts are signed

c) Firms produce and pay current wages; workers make saving decision and consume

Directed search with assets There is a continuum of labor markets indexed by the promised value to a worker denoted v and the assets of workers a. I assume that firms commit to never match with workers who have a different level of assets $\tilde{a} \neq a$. Since asset are public information, this assumption ensures that workers will only search in the labor market that was designed for them, and therefore that firms know the asset of the workers they match with when they design wage contracts. In section 5, I relax the assumption that assets are public information and evaluate whether, as a result, a worker with asset a would benefit by searching in markets indexed by $\tilde{a} \neq a$.

Every period, workers with asset a choose in which labor market v to search, and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets. Firms post vacancies in these markets and, only after they match, learn about their productivity x and $\overline{x}$.

Denote $\phi_u(v,a)$ and $\phi_e(v,a)$ the mass of unemployed and employed workers searching for a job and denote $\phi_f(v,a)$ the mass of vacancies posted by firms. Let κ denote the search intensity of employed workers relative to unemployed workers. In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)$ turns workers searching for a job and vacancies into matches. Define the job finding rate $\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that an unemployed worker finds a job, and the vacancy filling rate $\tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that a vacancy finds a worker. These probabilities are defined in the usual way as

$$\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_u + \kappa\phi_e}, \quad \tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_f}$$

Since these matching probabilities will depend on v and a in equilibrium, we can write them in short-hand notation as

$$\lambda_w(v,a) \equiv \tilde{\lambda}_w(\phi_u(v,a) + \kappa\phi_e(v,a), \phi_f(v,a)), \quad \lambda_f(v,a) \equiv \tilde{\lambda}_f(\phi_u(v,a) + \kappa\phi_e(v,a), \phi_f(v,a))$$

In equilibrium there will be an upper bound $\overline{v}$ on the set of active labor markets. For $v > \overline{v}$, the job finding rate is not defined because no firm post vacancies there. I extend this function by setting it to 0 for these values above $\overline{v}$. This implies that a worker can always choose not to search by selecting $v > \overline{v}$.

Unemployed workers Unemployed workers face a standard consumption-savings decision problem similar to Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023). They receive unemployment benefits b and choose how much to save and consume. They also choose in which labor market v to search. Given the job finding probability, $\lambda_w(v,a)$, the value of unemployed workers satisfies

$$\begin{aligned}
U(a_t) = \max_{c_t, a_{t+1}, v_{t+1}} \quad & u(c_t) + \beta\left[\lambda_w(v_{t+1}, a_{t+1})v_{t+1} + (1 - \lambda_w(v_{t+1}, a_{t+1}))U(a_{t+1})\right] \\
\text{s.t.} \quad & c_t \leq (1+r)a_t + b - a_{t+1} \\
& a_{t+1} \geq 0
\end{aligned}$$

In choosing in which labor market v to search, workers face the following standard trade-off: searching in a high-v labor market brings a higher value v conditional on a match, but it will turn out that these matches occur with lower probability because $\lambda_w(v,a)$ will decrease with the value v in equilibrium. The choice of search v also depends on their current assets a. A worker with higher assets search for jobs with higher values v and lower job finding rates.

Employed workers Employer workers also search for jobs. With probability $\kappa\lambda_w(v, a_t)$, they find a new match in market v when their current asset is a_t.

Existing matches break up and workers separate into unemployment with exogenous probability δ. I assume that firms cannot make any transfer to workers conditional on exogenous separations. As I explain in section 3, this assumption is critical because it implies that unemployment shocks are a source of risk for workers that firms cannot insure, and thus workers will accumulate precautionary savings against it. There are several ways to justify this assumption. First, if exogenous separations occur because of bankruptcy, then firms are not able to make severance payments to workers. Second, if firms have limited commitment, they have no incentives to make a payment to workers after an exogenous separation since the match ends. Finally, in the data the consumption of workers drops when they become unemployed, which shows that this is a source of risk that firms have not entirely insured away.

Contracts When firms and workers are first matched, they sign wage contracts. The contract is subject to two contracting frictions, which both play a critical role in the analysis. First, the worker search decision is private information, which leads to moral hazard. Second, workers have limited commitment in that they cannot commit to make a transfer back to firms after an EE transition. These two frictions are the reason why firms provide limited insurance to workers. In section 3, I show how firms manipulate assets strategically to circumvent these frictions.

I assume that the savings decision of workers a_{t+1} is public information, which means that it is contractible[2]. Productivity shocks $(\overline{x}, x_t)$ are also public information. Finally, I assume for simplicity that it is unfeasible for firms to make counteroffers to their workers when they receive outside job offers[3].

[2] When assets are private information, two issues arise with wage contracts. First, private information about savings decision leads to joint deviations where the worker saves less and searches for jobs with lower values v than what is recommended in the optimal contract. This is similar to the issue of hidden savings studied in Werning (2002) and Abraham and Pavoni (2008). Second, with directed search workers with assets a might deviate by searching for jobs in labor markets designed for workers with assets $\tilde{a} \neq a$. This is similar to the issue studied in Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023).

[3] This assumption can be formally justified as follows: counteroffers are private information to workers, and expire before workers can return to their current employers to negotiation for higher wages. These assumptions ensure that it is optimal for current employers not to respond to outside offers.

2.2 Optimal contracts

Following previous work on dynamic contracts, I write the contract recursively in terms of promised values and continuation values instead of histories of shocks[4]. Denote V_t the promised value of a worker at the start of the period. The state of a match at the beginning of the period are the worker promised value V_t, the assets of the worker a_t as well as the current productivity x_t.

The components of the contract at time t are the wage paid today, the savings decision of workers and a set of continuation values for each state tomorrow. Formally, these components are represented by the functions

$$w_t\left(V_t, a_t, x_t\right), \quad a_{t+1}\left(V_t, a_t, x_t\right) \quad \text{and} \quad V_{t+1}(V_t, a_t, x_t, x_{t+1})$$

A *contract* is a collection of these functions for all t. In the recursive formulation below, we write the components of the contract as $w_t, a_{t+1}, V_{t+1}(x_{t+1})$. It will be convenient to denote the continuation value of workers at the current job by

$$W_t \equiv u(c_t) + \beta \mathbb{E}_{x_{t+1}}\left[V_{t+1}(x_{t+1})|x_t\right]$$

This value is pinned down by the contract.

Worker value Given a contract, the worker chooses a search strategy to maximize the present value of utility. The value of a worker satisfies

$$V_t = \delta U(a_t) + (1-\delta)\max_{v_t}\left[\kappa\lambda_w(v_t, a_t)v_t + \left(1-\kappa\lambda_w(v_t, a_t)\right)W_t\right] \tag{1}$$

In (1), the first term is the continuation value of a worker who becomes unemployed at time t with assets a_t. The second term depends on the probability that a worker finds another job $\kappa\lambda_w(v_t, a_t)$ and on the value that the worker receives if an EE transition occurs v_t. A worker who does not find a new job receives the continuation value W_t.

Equation (1) shows how the assumptions of hidden search and limited commitment interact. First, moral hazard arises because the worker's search decision only depends on the surplus that workers get from EE transitions $v_t - W_t$, and not on the firm value. Thus, relative to the search policy that the worker chooses, the firm with a positive value would prefer the worker to search instead in markets with a higher value v and a lower job finding rate $\lambda_w(v, a)$ because it wants to retain the worker. Since the worker search

[4] I abstract from randomized contracts to keep notations simple.

decision v_t is private information, the firm cannot control it directly and instead influences the worker's decision by manipulating continuation values.

The limited commitment assumption implies that firms can only manipulate the continuation value of workers at the current job W_t and not after an EE transition v_t. When workers have access to assets a_t however, the firm has an additional tool to influence the worker's search decision since assets a_t influence the job finding rate $\lambda_w(v_t, a_t)$ of workers. Section 3 expands on this role that assets play, and shows that assets are in fact a substitute for the commitment power of workers.

Optimal contracts Denote the search policy $v(W_t, a_t)$, define the EE probability as

$$p(W_t, a_t) \equiv \kappa \lambda_w(v(W_t, a_t), a_t)$$

and define the worker expected surplus from EE transitions as

$$S(W_t, a_t) \equiv \kappa \lambda_w(v(W_t, a_t), a_t) \left(v(W_t, a_t) - W_t\right)$$

Finally, denote $\Pi(V_t, a_t, x_t)$ the present value of profits for a firm matched with a worker with promised value V_t and assets a_t when productivity is currently x_t. Taking as given the value of unemployment $U(a_t)$, the job finding rate $\lambda_w(v_t, a_t)$, and the search policy of workers $v(W_t, a_t)$, the value of a firm satisfies

$$\Pi(V_t, a_t, x_t) = \max_{w_t, c_t, a_{t+1}, V(x_{t+1})} (1-\delta)(1-p(W_t, a_t)) \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}} \left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t \right]}{1+r} \right)$$

subject to

$$\begin{aligned}
&\text{(PK)}: && V_t \leq \delta U(a_t) + (1-\delta)\left[W_t + S(W_t, a_t)\right] \\
&\text{(Budget)}: && c_t + a_{t+1} = (1+r)a_t + w_t \\
&\text{(BC)}: && a_{t+1} \geq 0
\end{aligned}$$

where $W_t = u(c_t) + \beta \mathbb{E}_{x_{t+1}} \left[V_{t+1}(x_{t+1}) | x_t \right]$.

The firm maximizes the present value of profits, where $(1-\delta)(1-p(W_t, a_t))$ is the probability that the worker remains within the current match this period. The first constraint (PK) is the promise keeping constraint, stating that the value the worker gets from the contract either at the current job, through unemployment or at future jobs must deliver at least the promised value V_t. The second constraint (Budget) is the budget constraint of the worker, and the third constraint (BC) is the borrowing constraint. We wrote the optimal contract taking as given the optimal search policy of workers, so the incentive compatibility constraint is implicit in the definition of $p(W_t, a_t)$ and $S(W_t, a_t)$.

Value of new matches Consider the value of a firm when it is just matched with a worker in market (v, a). The value from this match, denoted $\Pi_0(v, a_t)$, solves

$$\Pi_0(v, a_t) = \mathbb{E}_{\overline{x},x_0}\left[\max_{w_t,c_t,a_{t+1},V(x_{t+1})} x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1})|x_t\right]}{1+r}\right] \tag{2}$$

subject to

$$\text{(PK)}: \quad v \leq \mathbb{E}_{\overline{x},x_0}\left[u(c_t) + \beta \mathbb{E}_{x_{t+1}}\left[V_{t+1}(x_{t+1})|x_t\right]\right]$$

and the constraints (Budget) and (BC). All the choice variables are conditional on the realization of productivity $\overline{x}, x_0$. The firm fixed productivity is drawn from $\log \overline{x} \sim \mathcal{N}(0, \sigma^2_{\overline{x}})$ and the initial value of firm productivity is drawn from $\log x_0 \sim \mathcal{N}(\log \overline{x}, \sigma^2)$.

2.3 Equilibrium

Free entry Vacancy posting is subject to free entry. Firms have to pay a unit cost for posting a vacancy k so the free entry condition is

$$-k + \lambda_f(v, a)\Pi_0(v, a) \leq 0 \tag{3}$$

with equality for each active market (v, a).

Definition of an equilibrium An equilibrium is a set of value functions, policies and matching rates for each labor market (v, a) such that i) the unemployed worker policy maximizes the unemployment value, ii) the firm and employed worker policies satisfy the optimal contract, iii) the free entry condition is satisfied and iv) the job finding and vacancy filling rates are consistent with the matching function.

Denote the probability density function of the distribution of unemployed and employed workers by $\{D^u(a), D^e(V, a, x)\}$. In equilibrium, the laws of motion of distributions are satisfied given the policies.

3 The role of assets in wage contracts

I now characterize the optimal contract, with an emphasis on how assets alter the allocation relative to similar optimal wage contracts without assets (as in Shi, 2009) and relative to the unemployment insurance literature where assets play no role (as in Hopenhayn and Nicolini, 1997). I first show that in the optimal contract assets satisfy a pseudo Euler equation, and are used both to smooth the worker consumption and to influence her search decisions through wealth effects. I then show how, as a result, assets influence the degree of wage backloading in the optimal contract.

3.1 A pseudo Euler equation

I derive the optimality condition for assets, which shows that firms use them to smooth the consumption of workers and influence their search decision.

Combining the first-order condition for assets a_{t+1} and the envelope condition leads to an equation that depends on how the EE probability and the worker surplus from EE transitions change with assets, that is $p_a(W_{t+1}, a_{t+1})$ and $S_a(W_{t+1}, a_{t+1})$. These terms reflect the fact that assets influence the search decision of workers $v(W_{t+1}, a_{t+1})$ and the matching rate in each market $\lambda_w(v_{t+1}, a_{t+1})$.

To further simplify this equation, we need to understand how assets influence job finding rates and worker mobility decisions. For this, we turn to the free entry condition that connects the value of new matches Π_0 and the vacancy filling rate λ_f (and thus the job finding rate λ_w through the matching function). Notice first that $\Pi_0(v, a_{t+1}, x_{t+1}) = \Pi_0(v, 0, x_{t+1}) + (1+r)a_{t+1}$, so the value of new matches increases in the asset of the worker. Notice next that the firm value Π_0 is strictly decreasing in the worker promised value v from the envelope condition. Thus, firms are indifferent between matching with a worker in a market with high value and high assets, and in a market with lower value but also lower assets. This indifference is captured by the following equation, derived from the free entry condition,

$$\partial_a \lambda_w(v_{t+1}, a_{t+1}) = -\partial_v \lambda_w(v_{t+1}, a_{t+1})(1+r)u'(c_{t+1}^{EE})$$

where c_{t+1}^{EE} is the consumption of the worker during the first period after an EE transition. This equation states that in equilibrium increasing assets by 1% has the same effect on the EE probability than inducing workers to search in a market with $(1+r)u'(c_{t+1}^{EE})$% lower values.

Now, combine this equation with the optimality condition for search v to get

$$S_a(W_{t+1}, a_{t+1}) = \partial_a \lambda_w(v_{t+1}, a_{t+1})\left[v_{t+1} - W_{t+1}\right] = p(W_{t+1}, a_{t+1})(1+r)u'(c_{t+1}^{EE})$$

Intuitively, increasing assets by Δa makes workers more likely to match with another firm, and has a similar effect on the surplus from EE transitions than increasing consumption at the next job by $(1+r)\Delta a$. This equation shows why differentiating the promise keeping constraint with respect to assets a_t leads to a pseudo Euler equation.

With this result in mind, we can rewrite the optimality condition for assets a_{t+1} in the optimal contract as *a pseudo Euler equation*

$$\frac{u'(c_t)}{\beta(1+r)} \geq \delta u(c^u_{t+1}) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p_{t+1}u'(c^{EE}_{t+1}) + (1-p_{t+1})u'(c_{t+1})|x_t\right] - \mathcal{W}_t \tag{4}$$

where wealth effects on search are

$$\begin{aligned}\mathcal{W}_t \equiv \quad & (1-\delta)\frac{u'(c_t)}{\beta(1+r)}\mathbb{E}_{x_{t+1}}\Bigg[\left(u'(c_{t+1})p_W(W_{t+1},a_{t+1}) + \frac{p_a(W_{t+1},a_{t+1})}{1+r}\right) \\ & \qquad\qquad \times\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\Bigg]\end{aligned}$$

Equation (4) holds with equality when the borrowing constraint does not bind. This equation, derived in appendix A.3, shows that assets are used in the optimal contract to smooth the consumption of workers, and to influence their search decision.

In the next subsection, I show that the ability of firms to smooth the consumption of workers using assets dramatically changes the optimal contract. Before this, I briefly discuss the implications of the wealth effects on search.

Wealth effects on search In a directed search model, searching for a job is similar to selecting a lottery with payoff v and winning probability $\lambda_w(v,a)$. Whether workers select a lottery with high risk and high payoff or a lottery with low risk and low payoff depends on their preferences. With CRRA utility, workers with different levels of assets a select different lotteries. In particular, a worker with low asset selects a safer lottery in which jobs are easier to get but yield a lower value. How does this influence the contract? When the firm continuation value is positive, the firm wants to retain its worker and thus influences the search decision to reduce the EE rate. Therefore, relative to a model without wealth effects, the firm increases the asset of workers to make them select riskier lotteries, that is search in market with a higher value v and lower job finding rate $\lambda_w(v,a)$.

3.2 Implications for the path of wages

In the previous subsection I showed that firms use assets to smooth the consumption of workers. I now derive the implications for the path of wages, consumption and EE mobility in the optimal contract.

The starting point here is the following consumption growth condition

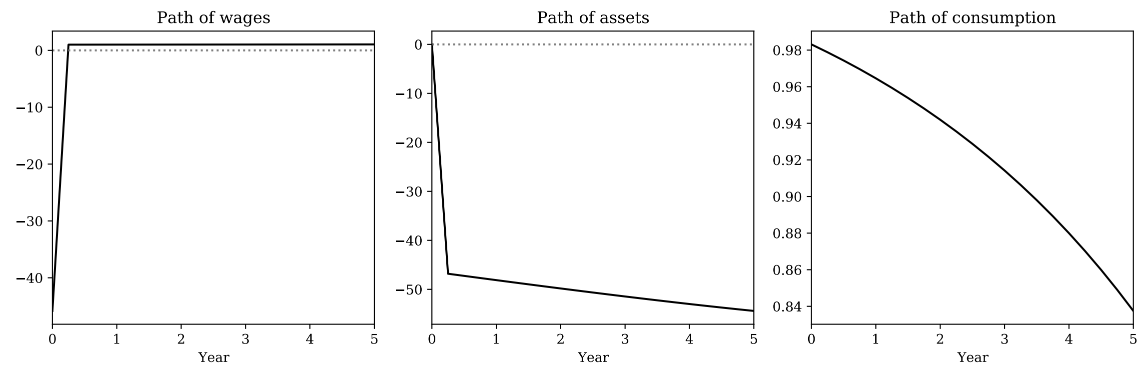
$$\frac{1}{u'(c_t)} - \frac{\beta(1+r)}{u'(c_{t-1})} = -\frac{p_W(W_t, a_t)}{1-p(W_t, a_t)} \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1})|x_t\right]}{1+r} \right) \tag{5}$$

which is derived in details in appendix A.2 from the optimality conditions of the contracting problem. This equation relates the growth rate of consumption, the worker EE probability p_W, p and the continuation value of the firm. This equation is similar to theorem 1 in Burdett and Coles (2003), lemma 3.2 in Shi (2009) and proposition 2 in Balke and Lamadon (2022), except that consumption c now replaces wages w on the left-hand side. When workers have no access to savings, $c = w$, equation (5) pins down the path of wages in the optimal contract and leads to the standard wage backloading result. Instead, when workers have access to savings, this equation must be combined with the pseudo Euler equation (4) to pin down the path of wages. I now describe how the interaction between wages, consumption and assets alters the standard wage backloading result.

Extreme wage backloading without unemployment risk I first consider the optimal contract when the separation rate δ is set to 0 so that workers face no unemployment risk. For simplicity, I also assume that workers face no borrowing constraint but the key insight holds even with it.

Figure 1 shows the optimal path of wages, assets and consumption for a newly hired worker from unemployment, starting with no assets $a_0 = 0$. In this specific example, the productivity of the firm happens to remain constant at $x_t = 1$ throughout. The optimal contract implies a large negative wage upon hiring, followed by much higher wages in subsequent periods. Meanwhile, consumption is much smoother and in this specific example falls over time. This is feasible because workers borrow substantially in the first period to finance their consumption. Thus, the optimal contract implies an extreme form of wage backloading.

Figure 1: Tenure profiles without unemployment risk and borrowing constraint



To understand why wages are so backloaded when workers face no unemployment risk, it is useful to remember the intuitions behind the standard wage backloading results when workers have no access to savings. Firms face a trade-off when they design the path of wages. On one hand, a firm with positive value want to backload wages because it makes workers search in markets with higher values and lower job finding rate. Thus, backloading helps profitable firms retain their workers. These forces are captured in the right-hand side of equation (5) when $w = c$. On the other hand, to backload wages is to backload consumption because workers have no access to savings. Thus, backloading wages too much would make contracts very unattractive to workers. This is captured by the left-hand side of equation (5) when $w = c$. The optimal degree of wage backloading balances the desire of firms to retain its workers, with the desire of workers for smooth consumption.

How does this trade-off change when workers have access to savings? Firms can backload wages significantly more and ensure that the consumption of workers is smooth by making them borrow or use their existing assets. This is the reason why the optimal contract, as shown in figure 1, implies an extreme form of backloading. When workers face a borrowing constraint, firms set wages so that workers use all their existing assets in the first period. Thus, we still get a stronger form of wages backloading relative to a model without assets.

Irrelevance of assets with worker commitment Figure 1 shows that introducing assets alters the optimal contract significantly. This results might be surprising in relation to the optimal unemployment insurance literature where assets, when they are public information, are irrelevant for the optimal contract. The next proposition shows that assets only play a role in my model without unemployment risk because they are a substitute for the limited commitment of workers.

Proposition 1. *Assume that workers face no unemployment risk $\delta = 0$ and no borrowing constraint and that productivity remains constant during the match, $x_t = x_0$. Then, the path of consumption and EE probability $\{c_t, p_t\}$ are identical in the model with limited worker commitment and assets and in the model with worker commitment, with or without assets.*

Proof. See appendix A.4 □

This result shows that introducing assets in the model with limited worker commitment is equivalent to assuming worker commitment, that is they can commit to transfers to their previous employer after an EE transition. In the unemployment insurance literature, at least since Hopenhayn and Nicolini (1997), it is common practice to assume that the government (the principal) can tax workers (the agent) when they find a job. This assumption is equivalent to assuming commitment on the side of workers. Since workers already have commitment, introducing assets does not change the allocation when they are public information[5].

The allocation may be identical with worker commitment or assets, but the implementations are very different. To understanding why, remember that because of moral hazard, firms want to manipulate the continuation values of workers both at their current jobs and after an EE transition to influence their search decision. When workers have commitment, firms can do this using transfers from workers after EE transitions. With limited commitment, transfers after EE transitions are no longer possible but firms can instead influence the continuation value of workers at new jobs with assets. Specifically, by making the worker start their new jobs with less assets, firms reduce their wealth and therefore their consumption permanently. Proposition 1 shows that in fact, using either transfers or assets, we can achieve exactly the same allocation.

This result also sheds light on why consumption and assets are decreasing over time in figure 1. In this particular example, the productivity of the current firm ($x = 1$) is lower than the average productivity of firms posting new vacancies. Therefore, the optimal contract is designed to induce workers to search for another job while at the same time insuring them against the risk of not finding that other job. This moral hazard is resolved by lowering the consumption of workers over time at the current job, and by lowering their consumption at the next job by reducing assets over time. This result is reminiscent of Hopenhayn and Nicolini (1997), where unemployment benefits fall and the tax at the next job rises with unemployment duration.

[5]One might wonder why the literature on optimal wage contracts has assumed worker limited commitment. The reason is that this assumption is not just realistic (bonding between firms and workers is forbidden by law) but also critical to obtain wages that increase with tenure, as in the data.

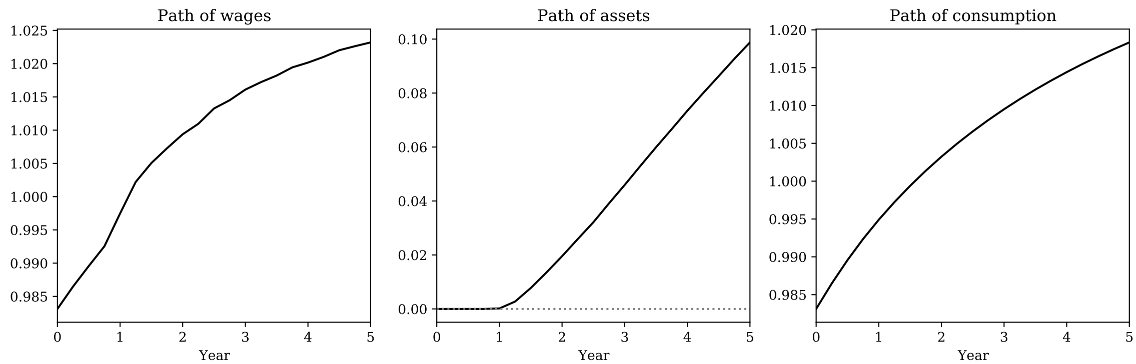
Partial wage backloading with unemployment risk In the model described in section 2, workers face positive unemployment risk ($\delta > 0$) that is uninsurable by firms, and borrowing constraints.

Figure 2 shows the optimal tenure profiles in the quantitative model described in section 2, for a newly hired worker from unemployment, starting with no assets $a_0 = 0$ and with a firm where productivity happens to remain constant at $x_t = 1$. Wages are now always positive and mildly increasing over time. Assets remain at the borrowing constraint for the first year of tenure, and then gradually increase over time. Finally, consumption increases over time instead of falling. Therefore, optimal contracts now feature partial wage backloading.

What explains the differences between figures 1 and 2? When workers face uninsurable unemployment risk, backloading wages too much and making workers borrow makes contracts very unattractive to risk-averse workers. If workers were to become unemployed, their income would fall and they would have to reduce their consumption drastically. Instead, workers would prefer to accumulate precautionary savings in anticipation of future unemployment shocks. Therefore, making workers borrow is costly because firms would have to pay workers much higher average wages to be able to attract them with such risky contracts.

The optimal contract features a trade-off between worker retention and precautionary savings. Backloading wages allows firms to retain workers, but frontloading wages allows them to attract workers by helping them to self insure against unemployment risk. Wages are more likely to be backloaded when the competition for workers is strong or the profits that firms generate are large. They are more likely to be front-loaded when unemployed risk is large because worker want to accumulate precautionary savings more. The borrowing constraint is critical because it makes unemployment riskier. Without it, workers would be able to smooth consumption almost perfectly if they become unemployed by borrowing against their future income.

Figure 2: Tenure profiles with unemployment risk



Over time, the level of assets for workers increases because the precautionary savings motive becomes stronger than the worker retention motive. As wages increase, the profits that firms generate fall, which makes the worker retention motive weaker. At the same time, the gap between the wage of the worker and unemployment benefit b increases, which makes the precautionary savings stronger. As a result, it becomes optimal for firms to increase the assets of workers.

Extreme wage frontloading without EE transitions Lastly, it is instructive to consider the case where workers cannot switch jobs ($\kappa = 0$). In this case, the optimal contract implies the opposite pattern: wages are extremely frontloaded. Firms pay workers a large initial wage, followed by much lower wages later on. As a result, a worker who becomes unemployed has enough assets to maintain consumption constant. Effectively, this worker has a marginal propensity to consume from unemployment shocks of zero. An implication is that introducing optimal wage contracts without worker mobility in a model with precautionary savings completely unravels the market incompleteness.

4 Quantitative analysis

In this section, I quantify the model using matched employer-employee data from France and measure how firms balance the need to retain their workers by backloading wages, with the need to insure workers against unemployment risk by frontloading wages. I then use the model to quantify how much insurance workers receive inside and outside the firm after a firm-level productivity shock.

4.1 Quantification

I quantify the model at quarterly frequency using matched employer-employee data from France between 2008 and 2019.

Data I combine annual data on firm balance sheet (FARE) with a panel of worker from social security data (DADS) containing 1/12th of the French labor force. I only keep in the sample workers with permanent full time contracts, and prime age workers (25-55 years old). I focus on private sector jobs in for-profit firms with at least 3 employees. The final sample contains approximately 530,000 workers and 130,000 firms per year.

Quantification strategy I quantify the model in two steps: first, I set some parameters externally; second, I infer the remaining model parameters by moment matching.

The model parameters set externally are the utility function, the foreign interest rate r and the matching function. The utility function is CRRA with coefficient $\gamma = 2$, following standard estimates from the macroeconomic literature. I set the interest rate to 4% annually. The matching function is Cobb-Douglas

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu} \phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). $B = 0.26$ is calibrated to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the job finding rate in my model[6].

The other model parameters are inferred by matching moments in the French data and in model-simulated data. Specifically, I simulate a panel of workers in the model and estimate the exact same set of moments in the model and in the data. The estimated parameters are the discount rate β, the vacancy posting cost k, the search efficiency on the job κ, the value of home production b, the exogenous separation rate δ, the persistence of productivity ρ_x, its volatility σ_x and the dispersion in productivity across matches $\sigma_{\overline{x}}$. These 8 parameters are estimated using 8 moments in the data.

Table 1 shows the moments used in the estimation and the parameter values. I use moments on labor market flows (UE, EE and EU rates) to discipline the mobility of workers across jobs and the risk that workers face from unemployment. I use the tenure profile (25-year cumulative change) of log residual wages and EE transitions to discipline optimal contracts. I use moments on firm productivity, measured as value added per worker, to discipline the risk that firms face and might transmit to workers. Finally, I target an

[6] I do not have vacancy data for France and thus cannot estimate the matching function directly.

Moments	Data	Model	Parameters	
MPC from EU shock	30%	32%	Discount factor $\beta(1+r)$	0.995
Quarterly UE rate	20%	20%	Cost of posting vacancy k	0.55
Annual EE rate	6%	6.1%	On-the-job search efficiency κ	0.3
Annual EU rate	6%	6.1%	Separation rate δ	0.017
25-year cumulative wage growth	10%	10.9%	Flow unemployment value b	0.75
25-year cumulative ΔEE	-10%	- 8.6%	Dispersion in match productivity $\sigma_{\overline{x}}$	0.15
sd. of firm productivity growth	0.3	0.3	Volatility of productivity σ_x	0.14
Annual persistence of productivity	0.81	0.81	Persistence of productivity ρ_x	0.9

Table 1: Targeted moments in data vs. model and parameters

estimate of marginal propensity to consume after unemployment shocks of 30% quarterly, a standard value in the literature on precautionary savings, to discipline the access of workers to financial markets.

Estimation results Two estimated parameters are worth discussing. First, the discount factor is such that $\beta(1+r) < 1$. This is standard in models with precautionary savings to ensure that workers do not accumulate an infinite amount of savings (Sotomayor, 1984, Chamberlain and Wilson, 2000). In the context of optimal contracts, this implies that the principal (the firm) and the agent (the worker) have different discount factors, and thus that the promised value of workers will drift over time. This can be seen from equation (5) when $\kappa = 0$. This equation shows that, absent EE transitions, the consumption of workers falls over time when $\beta(1+r) < 1$.

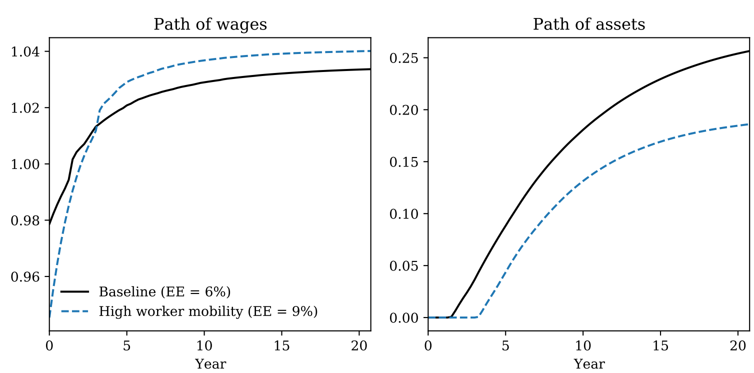
Second, the implied flow value of unemployment b is at 0.75, so approximately 70% of the average wage in the model. This rather high value is consistent with the high value of unemployment benefits in France. This parameter is mostly pinned down by the low tenure profile of wages (10%) for the following reason. The flow value of unemployment b determines how selective unemployed workers are to search for jobs. A high value of b implies that they will search for jobs with high starting wages. Since wages grow over time and eventually end up at the level of firm productivity, the starting value pins down the wage tenure profile. A low tenure profile thus indicates that workers started their jobs with relatively high wages, and thus that the value they get from unemployment is high. This result, together with the low separation rate into unemployment (6% annually) implies that workers face little unemployment risk and thus that the precautionary savings motive is moderate. However, this is partially compensated by the low EE transition rate (6% annually), which also implies a moderate worker retention motive.

4.2 Quantifying incentives vs. insurance

I now quantify the strength of the worker retention and precautionary savings motives, described in section 3, in three steps. I first evaluate how an increase in EE transitions, making the worker retention stronger, influences the tenure profile of wages and the ability of workers to accumulate precautionary savings. I then evaluate how a reduction in unemployment benefits influences the competition for workers and unemployment risk. Finally, I compare the profits that firms generate using optimal wage contracts relative to fixed-wage contracts.

Comparative statics: increase in EE transitions I generate an increase in EE transitions from 6% annually to 9% by increasing the search efficiency of workers on the job κ. Figure 3 shows the path of wages for a newly hired worker from unemployment in the baseline model and in the counterfactual. When the EE transition rate is higher, there is more competition for workers so firms choose to backload wages more to retain them. As a result, wages are lower during the first 3 years of tenure and workers accumulate less assets. Wages eventually rise higher because of the increased competition for workers. Quantitatively, the wage tenure profile increases by 3.2 percentage points, relative to the baseline of 10.9% reported in table 1.

Figure 3: Tenure profiles for different degrees of worker mobility



What does the increase in competition for workers imply for the ability of workers to self-insure? The right panel of figure 3 suggests that workers accumulate less precautionary savings against unemployment, but the tenure profile only offers a partial answer because workers are also more likely to switch jobs with higher wages. Thus, whether workers are on average better insured against unemployment risk is ambiguous. I find that assets are actually 20% higher in the counterfactual economy with high EE transitions than in the baseline, and as a result the consumption of unemployed workers is

	Young (0-10 years)	Old (10-40 years)
Assets with high mobility/baseline	-18.1%	15.7%
Consumption after unemp. shock with high mobility/baseline	-1.8%	1.0%

Table 2: Asset accumulation over the life cycle

2% higher. This result however masks important differences across workers. In particular, workers who do not have much labor market experience do not have the time to accumulate precautionary savings, and as a result face more unemployment risk.

To quantify this difference, I simulate the path of assets, wages and consumption of workers starting at $t = 0$ from unemployment with no assets. I then measure the asset and consumption of these workers after an unemployment shock at different times in the counterfactual with high EE transitions and in the basline. The results are reported in table 2. The first column, labeled young, reports the average assets and consumption during the first 10 years. The second column, labeled old, reports the same variables between the years 10 and 40. The results confirm that workers accumulate less assets early on when worker mobility is high, but more later on. The difference over time is quite sizable, as workers accumulate 18% less assets during the first 10 years, 16% more during the next 30 years and 20% more on average including later periods. As a result, their consumption after an unemployment shock is lower early on and larger later on relative to the baseline.

Comparative statics: reduction in unemployment benefits I reduce unemployment benefits from $b = 0.75$ to $b = 0.5$. A reduction in unemployment benefits influences wage contracts along two margins. First, it makes unemployed workers less selective when they search for jobs, and as a result they are more likely to start a job from unemployment with a lower value. Since the starting value of workers is lower, the job ladder lengthens and reinforces the competition for workers. Second, the fall in benefits increases unemployment risk and the desire of workers to accumulate precautionary savings.

Table 3 reports the results of this exercise. As expected from such a policy, the job finding rate increases by about 8 percentage points. Because the job ladder has lengthened, the EE transition rate increases slightly. Workers now face significantly more risk, as emphasized by the increase in the volatility of consumption from 8.6% to 10.9%. Part of this increase is due to the fall in consumption when workers loose their jobs. Here, unemployment falls by 21% when workers become unemployed, as opposed to 14% in the baseline. The novel result however is that consumption volatility also increases on

	UE rate	EE rate	sd(c)	Δc^{EU}	25-y Δ c	Pass-through c	Assets
Baseline ($b = 0.75$)	20%	6.1%	8.6%	-14%	8.2%	3.1%	1.1
Low benefits ($b = 0.5$)	27.8%	6.3%	10.9%	-21%	13.5%	3.2%	2

Table 3: Reducing unemployment benefits

the job. Specifically, the consumption of workers now increases on average by 13.5% on the job after 25 years of tenure, against 8.2% before, and the pass-through of productivity shocks to consumption increases very slightly. Finally, in response to the policy workers accumulate more precautionary savings to self-insure against the shock. Overall, reducing unemployment benefits makes worker consumption more volatile after an unemployment shock but also at their current jobs.

Comparison with fixed-wage contracts I quantify the importance of optimal wage contracts for firms by comparing them with fixed-wage contracts. The motivation behind this exercise is that optimal contracts are complex to implement, and it is natural to wonder whether they are really worth it and why. There are of course many simple contracts that one can use as benchmark, but I select fixed-wage contracts since they are the simplest ones and they are often used in the literature (e.g. in Burdett and Mortensen, 1998).

I perform this comparison by computing the loss to a firm that adopts fixed-wage contracts relative to optimal contracts. Specifically, I take as given the equilibrium where all firms post optimal wage contracts. I then assume that a single firm deviates and posts contracts with a constant wage, where the wage level is such that a worker would be indifferent between working for this firm or for a firm offering the optimal contract. Finally, I compute the profits that such a firm posting fixed-wage contracts would generate if it were to match with an unemployed worker with no initial asset.

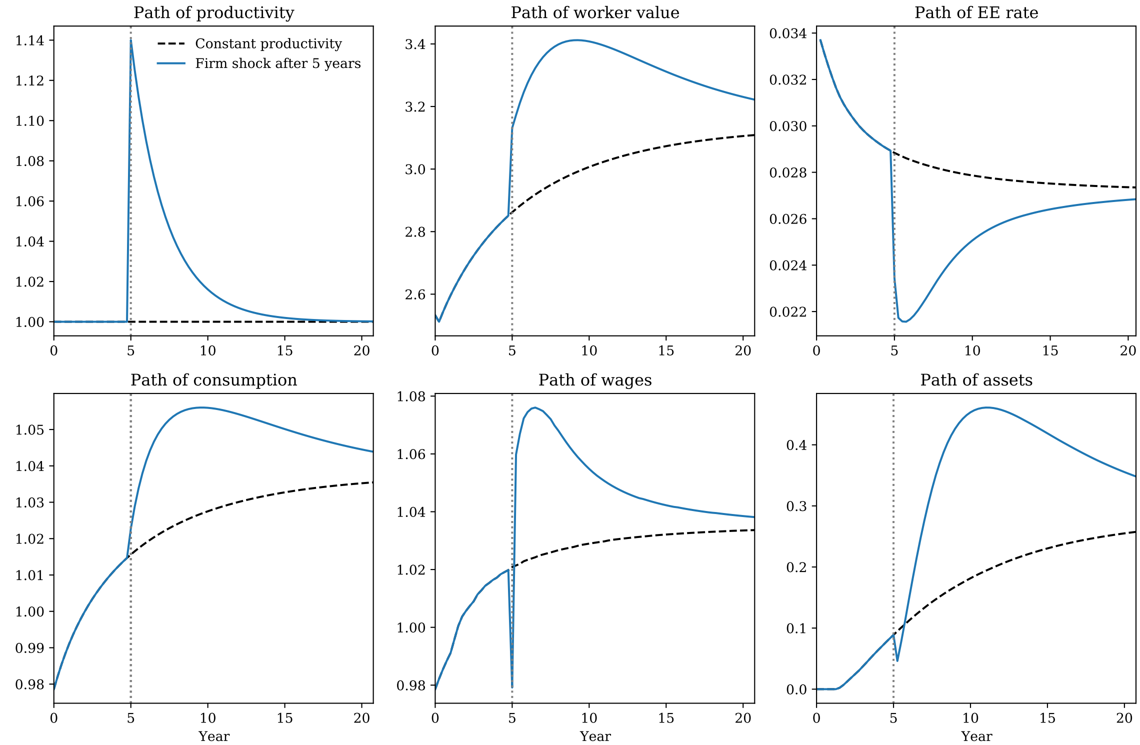
I find that adopting a fixed-wage contract is equivalent to reducing productivity by 7% permanently, a significant loss to profitability. This drop in profits occurs even though firms with fixed-wage contracts pay lower average wages since they provide much more stable consumption to workers. The reason for such a large difference in profits is that firms posting optimal wage contracts are able to retain workers more on average and especially when profits are high. For example, for a firm with the median level of fixed productivity $\overline{x} = 1$ the quarterly EE transition rate falls from 3.4 percent to 2.7 percent on average, and to 0.9 percent when profits are in the highest quintile.

4.3 Pass-through analysis: insurance inside vs. outside the firm

Finally, I revisit how firms pass productivity shocks through to workers. I am especially interested in comparing how much insurance workers receive inside the firm, through wage contracts, and outside the firm, through financial markets.

To analyze the pass-through, I first conduct the following experiment. Similar to figure 2, I compute the tenure profiles for a worker hired from unemployment with no asset at a firm with fixed productivity $\overline{x} = 1$ and where productivity happens to remain constant over time. I then compute the same tenure profiles except that after 5 years of tenure, the firm productivity unexpectedly increases by σ_x, and then falls back to its initial level with persistence ρ_x. The results of this exercise are shown in figure 3, where the dotted black lines represent the paths with constant productivity, and the solid blue lines represent the paths with the productivity shock. The difference between the two lines can be interpreted as the pass-through of productivity shocks. The top row shows that in response to the shock, firms choose to increase the worker value at the current job in order to reduce the EE transition rate. The bottom row shows how this increase in worker value is achieved. Firms make workers increase their consumption smoothly over time despite wages falling abruptly on impact and later increasing significantly. This is possible because workers initially reduce their savings. Eventually, the worker assets increase significantly, which implies that workers are better insured against unemployment shocks. Therefore, productivity shocks impact the consumption of workers at their current jobs but also their ability to self-insure against unemployment risk.

Figure 4: Pass-through of productivity shock



What explains these patterns of pass-through? Remember that risk-neutral firms want to insure risk-averse workers against shocks by keeping consumption constant, and the best way to do this is not to respond to shocks. However, not responding to shocks means that the EE transition probability of workers would be independent of firm productivity. The firm can instead increase profits by giving the worker a relatively high value when productivity and profits are high to induce them to stay, and a relatively lower value when productivity and profits are low. This is done by increasing the consumption of workers at the current jobs in response to a productivity shock. The size of the pass-through is pinned down by this trade-off between insurance and worker retention.

When workers do not have access to financial markets this path for consumption is implemented by increasing wages smoothly over time. By constrast, when workers can trade assets firms need not change wages and consumption the exact same way. Instead, firms choose to backload wages more in response to the positive shock, and wages actually fall on impact. Workers then use their savings to increase their consumption because they anticipate higher wages in the future. Why is it optimal to backload wages more in response to the shock and even reduce wages? As explained in section 3, the degree of wage backloading is the result of a trade-off between worker retention and precautionary savings. When productivity increases, the worker retention motive becomes more

	Pass-through to wages	Pass-through to consumption	MPC
Baseline model with assets	14.2%	3.1%	22%
Counterfactual: no assets	5.3%	5.3%	100%
Data	5%	n.a.	30%

Table 4: Pass-through of productivity shocks to wages and consumption

important because firms generate more profits and do not want to lose their workers, whereas the precautionary savings motive remains the same. As a result, firms choose to backload wages more despite exposing workers temporarily to more unemployment risk. This shows that firms take advantage of worker's access to financial markets when they respond to productivity shocks.

By how much do assets change the amount of insurance that workers receive against productivity shocks? To answer this question, I compare the pass-through of productivity shocks in the baseline model to a counterfactual where all parameters are kept the same but workers are not allowed to save. Table 4 reports the results of this exercise. The first column reports the pass-through to wages, which measures how much insurance workers receive from firms. The second column reports the pass-through to consumption, which measures how much insurance workers receive overall. The last column reports the marginal propensity to consume out of wage shocks. When workers have access to financial markets, the pass-through to wages increases by about 3 (from 5.3% to 14.2%), whereas the pass-through to consumption falls by about 2 (from 5.3% to 3.1%). The marginal propensity to consume falls from 100% in the counterfactual to 22% in the model with savings. Therefore, when workers have access to insurance outside firms, they receive significantly less insurance from firms but more insurance overall.

Table 4 also highlights a challenge to match the data. The model with assets does significantly better regarding the marginal propensity to consume than the counterfactual. An MPC of 22% is broadly consistent with existing empirical estimates from the literature. However, the model without savings does much better regarding the pass-through to wages.

5 Hidden assets

Until now, I have assumed that the asset of workers is public information to firms. This assumption implies that firms know the assets that workers have when they match, and that firms and workers can contract on the saving decision of workers when they are employed. These implications are not very realistic, so it is natural to wonder whether the allocation would be very different if assets were private information to workers.

Two issues arise when assets are private information, which have been partially studied in the literature. First, workers might choose to direct their search in markets (v, a) even though their current asset is not a. This is an issue because firms expect workers to have assets a when they design wage contracts. This has been studied with fixed-wage contracts in Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023), but not with optimal contracts. Second, the saving decision of employed might differ from the firm recommendation embedded in the contract. This issue has been extensively studied in the optimal unemployment insurance literature (e.g. Werning, 2002, Abraham and Pavoni, 2008).

In this section, I characterize the deviations of workers when assets are private information. Specifically, I ask what deviation would a worker follow if she is offered the contracts designed under the assumption that assets are public information, while in fact the worker can lie about its existing assets when it matches with a firm and about its saving decision during employment. I first show that with CARA utility, there is no profitable deviation for workers so that the allocation is identical with hidden assets and publicly observable assets. I then show that workers benefit from deviation when utility if CRRA because of wealth effects on search. Throughout, I will emphasize results that differ from those already derived in the literature.

5.1 Equivalence with CARA utility: no wealth effect on search

I first show that with CARA utility $u(c) = -\gamma \exp(-\gamma c)$, the optimal contract when assets are public is also optimal when assets are private information to workers. This result is important because it shows that the assumption that assets are public information is not that critical for the analysis. In particular, we can interpret the model as one where firms propose a set of contracts (wages conditional on history of shocks and tenure) to workers, and where workers select the contracts they prefer depending on their assets and choose to save and consume independently of firms.

Proposition 2. *Assume that utility is CARA and that workers face no borrowing constraint. Then, the equilibrium allocation with hidden assets is exactly identical than the allocation with publicly observable assets.*

Proof. See appendix A.5. □

This equivalence between private and public information arises because of the absence of wealth effect on search with CARA utility and it relies on two results. First, in the pseudo Euler equation 4 the terms capturing wealth effects is null, that is $\mathcal{W}_t = 0$. This result, extensively studied in the literature on optimal unemployment insurance, arises because two workers with the same wage contract but different levels of wealth are equally likely to switch job. Thus, the optimal contract with observable savings decision satisfies the Euler equation, and therefore it solves the optimal contract with hidden savings conditional on the initial asset of workers being known to firms.

Second, the absence of wealth effects means that workers have no incentive to report a different level of asset than their actual assets when they first match with firms. In particular, a worker with asset a never wants to under-report or over-report her assets by searching for a job in a market $(v, \tilde{a})$ where $\tilde{a} \neq a$. To see why, it is useful to assume that workers do not face any borrowing constraint first. In this case, it turns out that in any market (v_1, a_1) and (v_2, a_2) such that $v_1 \exp(\gamma r a_1) = v_2 \exp(\gamma r a_2)$, the job finding rate will be the same and workers will be offered the exact same contract. Thus, any deviation to a market $(\tilde{v}, \tilde{a})$ from (v, a) will yield the same value to workers than a deviation to a market $(\tilde{v} \exp(\gamma r(\tilde{a} - a)), a)$, i.e. a market with a different promised value but the true asset. But then, searching in such a market was already feasible for workers and was not optimal since (v) solves the search decision of workers in (1) conditional on their asset a. Therefore, workers would not benefit from deviating to a market where firms expect to meet workers with $\tilde{a}$ when their actual asset is a.

When workers face borrowing constraints, they too cannot benefit from reporting a level of asset $\tilde{a}$ different than their actual asset a. To see why, remember that firms take into account the worker's borrowing constraint when they design wage contract. When they expect it to bind, they do not backload wages as much because they know that workers cannot smooth consumption as much by consuming their existing assets. Thus, a worker with a lot of assets would receive a path of wages that is less backloaded by pretending to have very little assets. However, this worker would also receive lower average wages because firms generate lower expected profits in these markets. Overall, the worker is worse off. Conversely, a worker would not benefit by pretending to have more assets than she actually have since in this case she would enjoy higher average wages but her consumption would be more backloaded. One way to interpret this result is by remembering that assets are used in the optimal contract as a substitute for the commitment power of workers. Ex-ante, workers would be better off if they could commit to transfers after EE transitions. Thus, they have no incentive to under-report their assets.

5.2 Optimal deviations with CRRA

I now show that the optimal contract when assets are public information is no longer incentive compatible when assets are private information and utility is CRRA. Specifically, I first solve for the optimal contract assuming that firms observe the worker's assets perfectly. I then ask how a worker would choose to deviate if she is offered this set of wage contracts. This exercise does not characterize the equilibrium with hidden assets but it helps assess whether the assumption that assets are public information is critical.

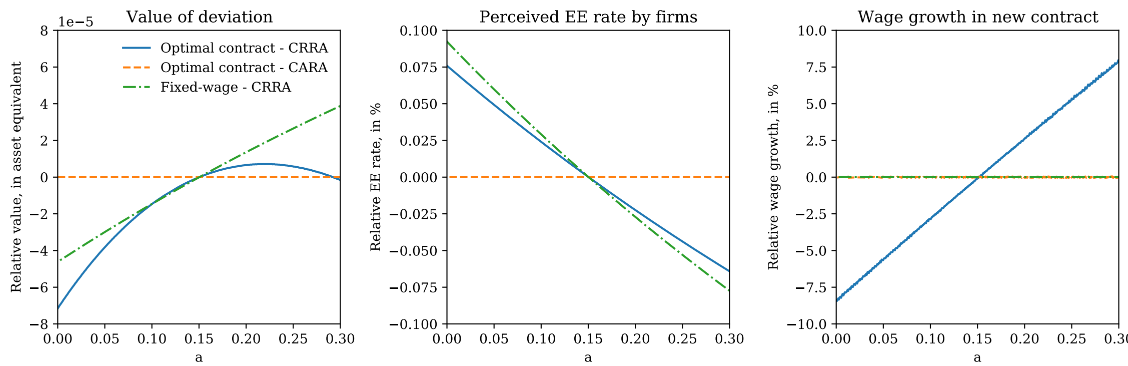
I first only briefly describe deviations in terms of savings that workers make while they are employed, since these have been extensively studied in the literature on optimal unemployment insurance. Equation (4) shows that when assets are public information, firms choose them to smooth the worker's consumption and to influence search through wealth effects $\mathcal{W}_t$. In particular, as explained in section 3.1, firms will make workers save more than the Euler equation would recommend when they want to retain them because this reduces the EE rate of workers. When the saving decision of workers is private information, the optimal deviation of workers is to follow their Euler equation, thus consume more today and save less. This deviation will make them more likely to switch jobs in the next period relative to what the firm would prefer.

I now describe the novel issue that arises in the context of wage contracts. Specifically, workers might deviate by searching in markets where the asset they are supposed to have $\tilde{a}$ is different than the asset they actually have a. The left panel of figure 5 illustrates this deviation by showing how much a worker with asset $a = 0.15$ would gain by searching in a market with $\tilde{a} \neq a$. For simplicity, the deviation is computed in a version of the model where workers only live for 2 periods, where they face no borrowing constraint and where productivity is constant across and within matches. I also compute contracts assuming that the saving decision is private information to stress the role that the initial asset of workers play[7]. The utility gains are measured in asset equivalent, that is how much additional assets would be needed to achieve the same gain than the deviation. The blue line shows that workers benefit by searching in markets with higher assets, thus pretending to be wealthier. In this example, the optimal deviation for a worker with asset $a = 0.15$ is to search in markets indexed by $\tilde{a} = 0.22$. By contrast, the orange line confirms that workers do not benefit by deviating when utility is CARA. Finally, the green line shows the value of a deviation with CRRA utility but when firms are restricted to offer fixed-wage contracts, as in Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023). In this case, workers also benefit from searching in markets with higher assets $\tilde{a}$

[7] I solve the optimal contract with hidden savings using the first-order approach, that is adding the Euler equation as a constraint on the optimal contract.

but the value from deviations is monotonically increasing in $\tilde{a}$.

Figure 5: Optimal deviation with hidden assets



What accounts for these differences? The middle panel of figure 5 shows why workers benefits by deviating upward when utility is CRRA. It shows the EE rate of workers with different assets *a keeping the wage contract constant*, relative to the EE rate of workers with $a = 0.15$. By keeping the wage contract constant, we can evaluate how firms perceive the likelihood that they can retain workers as a function of the assets of workers. The figure confirms that workers are less likely to leave when their initial assets are high because of wealth effects on search: relatively rich workers search in markets with a relatively high value and low job finding rate. Because workers are perceived by firms as less likely to leave, the expected profit from matches rises. Because of the free entry condition, workers end up receiving higher average wages in these markets. This is the reason why workers benefit by searching in markets designed for workers with higher assets.

The right panel of figure 5 shows why the gains from deviating are not monotonic with optimal contract, whereas they are with fixed-wage contracts. Specifically, the right panel shows the growth rate of wages that workers receive in markets with different assets a. Because workers are perceived to be less risk-averse when they report higher initial assets a, firms optimally choose to backload wages more. This makes these contracts relatively unattractive to workers with lower assets because they would prefer consumption to be smoother over time. As a result, workers choose to deviate upward but not too much. By contrast, when workers are offered fixed-wage contracts, they do not face this cost of deviating by assumption, and therefore the benefit from reporting higher assets is always increasing.

Taken together, these results show that the optimal contract described in section 3 and 4 is not incentive compatible when assets are private information. However, deviations by workers are bounded when contracts are optimal and the gains from deviating appear to be very small in this 2-period example. This is suggestive that the optimal contract might not be very different if firms took into account the fact that assets were private information to workers[8].

6 Conclusion

This paper studies the insurance that workers receive against firm-level productivity shocks when there are two competing forms of insurance available: insurance inside the firm, through wage contracts, and outside the firm, through non-contingent assets. I build a new model with optimal contracts and assets, and find that assets play a dual role in the contract. They allow firms to backload wages more, thus substituting for the lack of commitment by workers, and are used by workers to self-insure against unemployment risk. The optimal contract balances these forces and features a path of wages and worker mobility across jobs that is consistent with the data. I find that firms pass productivity shocks through to wages three times more and to consumption two times less relative to a model where workers are not allowed to trade risk-free bonds. Thus, when workers have access to insurance outside the firm, firms provide less insurance to workers but workers receive more insurance overall.

[8]This is reminiscent of Shimer and Werning (2008) who find that in the context of unemployment insurance, the allocation with hidden savings and CRRA utility is quantitatively almost identical to the allocation with observable savings.

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Appendix

Contents

A Model appendix

A.1 Optimality conditions

The optimal contract solves

$$\Pi(V_t, a_t, x_t) = \max_{w_t, c_t, a_{t+1}, W_t, V(x_{t+1})} (1-\delta)(1-p(W_t, a_t)) \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t\right]}{1+r} \right)$$

subject to

$$\begin{aligned}
&V_t \leq \delta U(a_t) + (1-\delta)\left[W_t + S(W_t, a_t)\right] && [\eta_t] \\
&W_t = u(c_t) + \beta \mathbb{E}_{x_{t+1}}\left[V(x_{t+1}) | x_t\right] && [\lambda_t] \\
&c_t + a_{t+1} = (1+r)a_t + w_t && [\mu_t] \\
&a_{t+1} \geq 0 && [\zeta_t]
\end{aligned}$$

The optimality conditions are

$$\begin{aligned}
w_t: &\quad (1-\delta)(1-p(W_t, a_t)) = \mu_t \\
c_t: &\quad \lambda_t u'(c_t) = \mu_t \\
V(x_{t+1}): &\quad (1-\delta)(1-p(W_t, a_t))(1+r)^{-1}\Pi_V(V(x_{t+1}), a_{t+1}, x_{t+1}) + \beta\lambda_t = 0 \\
W_t: &\quad -p_W(W_t, a_t)\left(x_t - w_t + \tfrac{\mathbb{E}_{x_{t+1}}[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1})|x_t]}{1+r}\right) + \eta_t(1-p(W_t, a_t)) - \tfrac{\lambda_t}{1-\delta} = 0 \\
a_{t+1}: &\quad (1-\delta)(1-p(W_t, a_t))(1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi_a(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t\right] = \mu_t - \zeta_t
\end{aligned}$$

and the envelope conditions are

$$\begin{aligned}
V_t: &\quad \Pi_V(V_t, a_t, x_t) = -\eta_t \\
a_t: &\quad \Pi_a(V_t, a_t, x_t) = -(1-\delta)p_a(W_t, a_t)\left(x_t - w_t + (1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t\right]\right) \\
&\qquad + \eta_t\left(\delta U'(a_t) + (1-\delta)S_a(W_t, a_t)\right) + \mu_t(1+r)
\end{aligned}$$

A.2 Consumption growth condition

We first derive the consumption growth condition (5).

Consider the optimality conditions with respect to $V(x_{t+1})$ and W_t in the optimal contract, and the envelope condition with respect to V_t to get

$$\eta_t = \beta^{-1}(1+r)^{-1}\eta_{t+1} + \frac{p_W(W_t, a_t)}{1 - p(W_t, a_t)} \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}} \left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t \right]}{1+r} \right)$$

Now combine the first-order conditions for c_t, w_t and $V(x_{t+1})$ and the envelope condition for V_t to get

$$\eta_{t+1} = \frac{\beta(1+r)}{u'(c_t)} \tag{A.1}$$

and therefore

$$\frac{\beta(1+r)}{u'(c_{t-1})} = \frac{1}{u'(c_t)} + \frac{p_W(W_t, a_t)}{1 - p(W_t, a_t)} \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}} \left[\Pi(V(x_{t+1}), a_{t+1}, x_{t+1}) | x_t \right]}{1+r} \right)$$

which is equation (5).

A.3 Pseudo Euler equation

We derive equation (4).

First, combine the first-order condition for a_{t+1} with the envelope condition for a_t to get

$$\begin{aligned} 1+r \geq \mathbb{E}_{x_{t+1}} &\left[\eta_{t+1} \left(\delta U'(a_{t+1}) + (1-\delta) S_a(W_{t+1}, a_{t+1}) \right) + (1-\delta)(1 - p(W_{t+1}, a_{t+1}))(1+r) | x_t \right] \\ &- \mathbb{E}_{x_{t+1}} \left[(1-\delta) p_a(W_{t+1}, a_{t+1}) \left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}} [\Pi(V(x_{t+2}), a_{t+2}, x_{t+2}) | x_{t+1}]}{1+r} \right) | x_t \right] \end{aligned} \tag{A.2}$$

with equality if the borrowing constraint does not bind.

Next, combine the first-order condition for c_t and the envelope condition of the unemployed workers to get

$$U'(a_t) = (1+r) u'(c_t^u) \tag{A.3}$$

where c_t^u denotes the consumption of the unemployed worker.

Finally, we need to derive an expression for $S_a(W_{t+1}, a_{t+1})$. From the envelope condition on the search problem of workers (1), we get

$$S_a(W_t, a_t) \equiv \kappa \left(v(W_t, a_t) - W_t \right) \partial_a \lambda_w(v(W_t, a_t), a_t)$$

We now derive an expression for $\partial_a \lambda_w(v(W_t, a_t), a_t)$ from the free entry condition of firms. Combining the first-order conditions and envelope conditions in the problem of new entrants (2) gives

$$\partial_v \Pi_0(v, a_t) = -\frac{1}{u'(c_t)}$$

$$\partial_a \Pi_0(v, a_t) = 1 + r$$

Now consider the free entry condition

$$\lambda_f(v, a) = \frac{k}{\Pi_0(v, a)}$$

Differentiating this expression with respect to v and a gives

$$\partial_a \lambda_f(v,a) = -\lambda_f(v,a)\frac{1+r}{\Pi_0(v,a)}$$

$$\partial_v \lambda_f(v,a) = \lambda_f(v,a)\frac{1}{\Pi_0(v,a)u'(c_t)}$$

Taking the ratio gives

$$\partial_a \lambda_f(v,a) = -\partial_v \lambda_f(v,a)(1+r)u'(c_t)$$

With a constant returns to scale matching function, we can express the job finding rate as $\lambda_w(v,a) = f(\lambda_f(v,a'))$. Therefore,

$$\partial_a \lambda_w(v,a) = -\partial_v \lambda_w(v,a)(1+r)u'(c_t)$$

This expression shows that it is equivalent for the job finding rate of workers to reduce the value of the labor market by $(1+r)u'(c_t)$ or to increase the asset of workers by 1%. We can use this expression to rewrite $S_a(W_t,a_t)$ as

$$S_a(W_t,a_t) = -(1+r)u'(c_t^{EE})\kappa\left(v(W_t,a_t) - W_t\right)\partial_v \lambda_w(v(W_t,a_t),a_t)$$

where c_t^{EE} is the consumption of the worker during the first period after an EE transition. We can further simplify this term using the first-order condition of the search problem

$$\lambda_w(v_t,a_t) + \partial_v \lambda_w(v_t,a_t)\left(v_t - W_t\right) = 0$$

and get

$$S_a(W_t,a_t) = (1+r)u'(c_t^{EE})\kappa\lambda_w(v(W_t,a_t),a_t) = (1+r)u'(c_t^{EE})p(W_t,a_t) \tag{A.4}$$

Combine equations (A.2), (A.1), (A.3), and (A.4), to get

$$\begin{aligned} u'(c_t) \geq\ & \mathbb{E}_{x_{t+1}}\left[\beta(1+r)\left(\delta u'(c_{t+1}^u) + (1-\delta)p(W_{t+1},a_{t+1})u'(c_{t+1}^{EE})\right) + (1-\delta)(1-p(W_{t+1},a_{t+1}))u'(c_t)|x_t\right] \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\frac{p_a(W_{t+1},a_{t+1})}{1+r}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

For the final step, rewrite the consumption growth condition (5) evaluated at $t+1$ as

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) - u'(c_t)u'(c_{t+1})\frac{p_W(W_{t+1},a_t)}{1-p(W_{t+1},a_t)}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}\left[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}\right]}{1+r}\right)$$

We can replace $u'(c_t)$ on the right-hand side by this expression and get

$$\begin{aligned} u'(c_t) \geq\ & \beta(1+r)\left(\delta u'(c_{t+1}^u) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p(W_{t+1},a_{t+1})u'(c_{t+1}^{EE}) + (1-p(W_{t+1},a_{t+1}))u'(c_{t+1})|x_t\right]\right) \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\left(u'(c_{t+1})p_W(W_{t+1},a_t) + \frac{p_a(W_{t+1},a_{t+1})}{1+r}\right)\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

which is equation (4).

A.4 Proof of proposition 1

I first describe the environment with worker commitment and state the contracting problem. I then show that assets are redundant with worker commitment, in that any allocation can be implemented with $a_t = 0$, which is a result that is well known in the optimal unemployment insurance literature. Finally, I prove proposition 1, which states that with limited worker commitment and assets the allocation is the same than with worker commitment.

Environment The search decision of workers is still private information but workers can now commit to a transfer to their previous employers after an EE transition. I assume that transfers can only be made when the worker leaves the firm, and not later.

Labor markets can still be indexed by the value that workers will receive v and by the asset that workers bring into the match a. However, the asset a is now net of the transfer that was made to the previous employer.

I focus on the case where workers do not face any borrowing constraint and where productivity is constant during matches, and I briefly explain why the equivalence breaks down when we relax these conditions at the end of the section.

Optimal contract Denote by $(1+r)\tau_t$ the transfer from workers to firm in the event of an EE transition. The search policy of workers now solves

$$v_t \in \arg\max_{\tilde{v}_t} \left[\kappa\lambda_w(\tilde{v}_t, a_t - \tau_t)\tilde{v}_t + (1 - \kappa\lambda_w(\tilde{v}_t, a_t - \tau_t))\, W_t\right]$$

where a_t represents the asset holdings of workers before the EE transition. We denote the search policy by $v(W_t, a_t - \tau_t)$. As before, we can define the EE probability and the surplus from EE transitions for workers as

$$p(W_t, a_t - \tau_t) \equiv \kappa\lambda_w(v(W_t, a_t - \tau_t), a_t - \tau_t)$$

$$S(W_t, a_t - \tau_t) \equiv \kappa\lambda_w(v(W_t, a_t - \tau_t), a_t - \tau_t)\,(v(W_t, a_t - \tau_t) - W_t)$$

The optimal contract now solves

$$\Pi(V_t, a_t, x_0) = \max_{w_t, c_t, a_{t+1}, W_t, V_{t+1}, \tau_t} (1 - p(W_t, a_t - \tau_t))\left(x_0 - w_t + \frac{\Pi(V_{t+1}, a_{t+1}, x_0)}{1+r}\right) + p(W_t, a_t - \tau_t)(1+r)\tau_t$$

subject to

$$\begin{aligned} &V_t \le W_t + S(W_t, a_t - \tau_t) && [\eta_t] \\ &W_t = u(c_t) + \beta V_{t+1} && [\lambda_t] \\ &c_t + a_{t+1} = (1+r)a_t + w_t && [\mu_t] \end{aligned}$$

and the value of new matches is still given by (2).

Assets are redundant with worker commitment Consider any allocation that solves the optimal contract with a path for assets a_{t+1}. We can construct a new path wages w_t and transfers τ_t such that the allocations c_t, p_t remain the same and still solve the optimal contract.

Define the new path of transfers as $\tilde{\tau}_t = \tau_t - a_t$ and the new path of wages as $\tilde{w}_t = (1+r)a_t + w_t - a_{t+1}$ for all periods except the first, and $\tilde{w}_t = w_t - a_{t+1}$ for the first period. Then, the firm value becomes

$$\Pi(V_t, a_t, x_0) = (1 - p(W_t, -\tilde{\tau}_t))\left(x_0 - \tilde{w}_t + (1+r)a_t - a_{t+1} + \frac{\Pi(V_{t+1}, a_{t+1}, x_0)}{1+r}\right) + p(W_t, -\tilde{\tau}_t)(1+r)(\tilde{\tau}_t + a_t)$$

Now conjecture that $\Pi(V_t, a_t, x_t) = \Pi(V_t, 0, x_t) + (1+r)a_t$. Use this guess to replace the firm value at $t+1$ and obtain

$$\Pi(V_t, a_t, x_0) = (1+r)a_t + (1 - p(W_t, -\tilde{\tau}_t)) \left(x_0 - \tilde{w}_t + \frac{\Pi(V_{t+1}, 0, x_0)}{1+r} \right) + p(W_t, -\tilde{\tau}_t)(1+r)\tilde{\tau}_t$$

which verifies our guess. We can now plug this in the problem of new entrants (2) to verify that the firm value is the same in the first period

$$\Pi_0(v, a_t) = \mathbb{E}_{\bar{x}, x_0} \left[x_0 - \tilde{w}_t + \frac{\Pi(V_{t+1}, 0, x_0)}{1+r} \right]$$

Therefore, the same firm value can be achieved with $a_{t+1} = 0$ and the paths $\tilde{w}_t, \tilde{\tau}_t$. Since these paths were constructed to maintain consumption the same, the constraints on the optimal contracts are still satisfied so the new contract with no savings solves the optimal contract.

Proof of proposition 1 I now turn to the main proof, showing that with limited worker commitment and assets, we can achieve the same allocation than with worker commitment.

Consider the optimal allocation c_t, p_t under commitment, achieved with some path of transfers τ_t. Without loss of generality, assume that $a_{t+1} = 0$ in this allocation. Limited worker commitment imposes that $\tau_t = 0$ for all t. To prove the equivalence, we construct new paths for assets and wages$\tilde{w}_t, \tilde{a}_{t+1}$ that provide firms with the same profits and maintain the same consumption c_t of workers. Since the allocation$c_t, \tilde{w}_t, \tilde{a}_{t+1}$ delivers the same profit to firms and satisfy the constraints with worker commitment, it solves the relaxed problem without the constraint $\tau_t = 0$. Since this allocation is also feasible with the constraint $\tau_t = 0$, it must solve the optimal contract with limited worker commitment.

Construct $\tilde{w}_t, \tilde{a}_{t+1}$ for all t using

$$\tilde{a}_{t+1} = -\tau_{t+1}$$
$$\tilde{w}_t = w_t - (1+r)\tilde{a}_t + \tilde{a}_{t+1}$$

with $\tilde{a}_t = 0$ in the first period. All paths are contingent on the realization of productivity in the first period x_0.

These paths ensure that the consumption of the worker remains the same at all date. It follows that V_t, W_t,$S(W_t, \tilde{a}_t)$ and $p(W_t, \tilde{a}_t)$ also remain the same so the constraints are still satisfied.

Now consider the firm value with these paths, denoted $\tilde{\Pi}_t$. It satisfies

$$\begin{aligned}
\tilde{\Pi}_t &= (1 - p(W_t, \tilde{a}_t)) \left(x_t - \tilde{w}_t + (1+r)^{-1}\tilde{\Pi}_{t+1} \right) \\
&= (1 - p(W_t, -\tau_t)) \left(x_t - w_t + (1+r)\tilde{a}_t - \tilde{a}_{t+1} + (1+r)^{-1}\tilde{\Pi}_{t+1} \right) \\
&= (1+r)\tilde{a}_t + (1 - p(W_t, -\tau_t)) \left(x_t - w_t - \tilde{a}_{t+1} + (1+r)^{-1}\tilde{\Pi}_{t+1} \right) + p(W_t, -\tau_t)(1+r)\tau_t
\end{aligned}$$

Now guess and verify that $\tilde{\Pi}_t = \Pi(V_t, 0, x_0) + (1+r)\tilde{a}_t$. Going back to the first period, we get

$$\begin{aligned}
\tilde{\Pi}_t = \;& \mathbb{E}_{\bar{x}, x_0} \left[x_0 - \tilde{w}_t + \tfrac{\tilde{\Pi}_{t+1}}{1+r} \right] \\
= \;& \mathbb{E}_{\bar{x}, x_0} \left[x_0 - w_t - \tilde{a}_{t+1} + \tfrac{\Pi(V_{t+1}, 0, x_0) + (1+r)\tilde{a}_{t+1}}{1+r} \right] \\
= \;& \mathbb{E}_{\bar{x}, x_0} \left[x_0 - w_t + \tfrac{\Pi(V_{t+1}, 0, x_0)}{1+r} \right] \\
= \;& \Pi_0(v, a)
\end{aligned}$$

This shows that the initial profit that firms generate from the optimal contract is the same with commitment, or limited commitment and assets. This is true even if the paths of profits in later periods differ. This concludes the proof.

Why the equivalence breaks down with borrowing constraints or productivity shocks In the example of figure 1, the optimal contract makes the worker borrow to fund a negative wage payments at $t = 0$. With borrowing constraints, this allocation is no longer feasible and the contract cannot implement the allocation with worker commitment. Thus, the equivalence breaks down.

When productivity varies during the match, the equivalence breaks down because with limited commitment, assets are non-contingent on the realization of productivity whereas with worker commitment, transfers are contingent on the realization of productivity.

Relation to the optimal unemployment insurance literature To the best of my knowledge, the equivalence between commitment and limited commitment with assets was not formally stated and proven before. However, it was implicit in Werning (2002) who studies the implications of hidden savings for optimal unemployment insurance. Specifically, Werning (2002) focuses his analysis on unemployment insurance policies that are implemented without taxing workers when they find a job, i.e. without worker commitment. This is without loss of generality because workers can save and thus restricting attention to such policies has no bearing on the path of consumption and job finding rate for workers.

A.5 Proof of proposition 2

The proof has two steps. First, we show that conditional on an initial value for worker assets a, the optimal allocation of the contract when assets are public information also solves the optimal contract when assets are private information. Second, we show that workers do not benefit by searching in markets indexed by assets $\tilde{a}$ when their actual asset is a. Together, these two results show that the optimal contract with public information is also optimal if assets were private information.

Details coming soon.

B Data appendix

I use administrative data provided by the CASD in France between 2008 and 2019. My analysis relies on two main files:

a) the panel version of the “DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) “FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms (“Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level ("UL"), and several legal units can belong to the same firm. Since I want to measure EE transitions across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the "Entreprise profilée" ("EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years and for firms with at least 3 employees (in the panel or not). I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.5}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying EE transitions is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job transition. An EE transition occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as EE transitions if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of EE transitions with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make an EE transition experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job transitions with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

When a worker separates from their previous jobs and does not make an EE transition, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. transition from private sector to public sector), I do not count it either as an EE transition nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.