Insurance Inside and Outside the Firm

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Abstract

This paper presents a new equilibrium model with optimal wage contracts, assets, and search frictions. In this model, firms take into account workers' access to financial markets when they design how wages change with tenure and after productivity shocks. The model is consistent with empirical evidence showing that wealthy workers experience higher average wage growth and are more exposed to firm-level productivity shocks than poor workers. The insurance that workers receive outside the firm significantly crowds out the insurance that they receive inside the firm. Specifically, firms provide relatively less insurance to wealthy workers because these workers can self-insure better. Wealthy workers are also matched with more productive firms and receive higher average wages precisely because firms provide less insurance to them. The model has novel implications for public policies that improve the ability of workers to self insure, such as relaxing borrowing constraints.

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1 Introduction

What determines the growth and volatility of worker earnings in the labor market? How can workers absorb fluctuations in their earnings to smooth their consumption? The existing literature has tackled this issue from two distinct perspectives. The first is to consider that firms influence the consumption of workers through the design of optimal wage contracts (Baily, 1974, Azariadis, 1975, Thomas and Worrall, 1988, Burdett and Coles, 2003, Shi, 2009, Menzio and Shi, 2010, Balke and Lamadon, 2022). In these models, workers receive insurance inside the firm when they receive wages that are stable over time and across states of the world. The second perspective is to consider that workers use their own assets to smooth consumption over time and in response to shocks (Bewley, 1977, İmrohoroğlu, 1989, Huggett, 1993, Aiyagari, 1994, Gourinchas and Parker, 2002, Kaplan and Violante, 2014). In these models, workers receive insurance outside the firm because they can trade risk-free bonds subject to borrowing constraints.

Despite decades of research on these issues, there has been no attempt to bring these two perspectives together. The existing models on optimal wage contracts make the stark assumption that workers have no access to financial markets. As a result, firms are the only source of insurance so they have a strong desire to smooth the earnings of workers. These models thus potentially overstate the role of firms as insurance provider. Besides, they cannot account for empirical evidence that wage growth and volatility depend on the wealth of workers (Guiso, Pistaferri and Schivardi, 2012, Fagereng, Guiso and Pistaferri, 2018, Halvorsen, Ozkan and Salgado, 2022). By contrast, the models used in the literature on consumption smoothing make the stark assumption that wage contracts are exogenous. As a result, these models are subject to the Lucas critique (Lucas, 1976) in that the growth and volatility of wages do not respond to changes in policy. Thus, we know very little about the interaction between the insurance that workers receive *inside the firm*, through wage contracts, and *outside the firm*, through financial markets.

This paper builds a new equilibrium model with optimal wage contracts, assets, and search frictions, which brings together the dynamic wage contracting model of Menzio and Shi (2010) with the canonical precautionary savings models in the tradition of Bewley (1977). In this model, firms take into account workers' access to financial markets when they design how wages change with tenure and in response to productivity shocks. I first characterize the optimal contract and then estimate the model using administrative data from France. The model is consistent with empirical evidence showing that wealthy workers experience higher average wage growth and are more exposed to firm-level productivity shocks than poor workers. I find that the insurance that workers receive outside the firm, by trading risk-free bonds, significantly crowds out the insurance that workers receive inside the firm, through wage contracts. Specifically, workers with more assets experience higher wage growth and are more exposed to firm productivity shocks because they can smooth their consumption better. However, despite having more volatile earnings these wealthy workers enjoy a consumption that is more stable over time and less volatile relative to workers with little assets. Thus, wealthy workers receive less insurance inside the firm but more insurance outside the firm and more insurance overall. Besides, wealthy workers are matched with more productive firms and receive higher average wages precisely because firms can optimize worker retention better when they do not provide as much insurance to workers. The model has novel implications for public policies that improve workers' ability to self insure, such as relaxing borrowing constraints. Specifically, enabling workers to borrow reduces the growth rate and volatility of their consumption despite increasing that of their income, and it increases allocative efficiency in the sense that workers are matched with more productive firms and receive higher average wages. This policy is especially effective for relatively poor workers, so it does not only improve efficiency but also reduces income inequality.

The model features risk-averse workers, risk-neutral firms and dynamic wage contracts with directed search. Workers move between employment and unemployment, and can switch jobs. They can trade non-contingent assets subject to a borrowing constraint. Firms select their technology before matching with workers and face firm-level productivity shocks. Firms and workers also face exogenous unemployment shocks. Firms post wage contracts, which specify how wages change over time and in response to shocks. Contracts are subject to two sets of frictions. First, the search decision of workers is private information, which leads to moral hazard. Second, workers and firms cannot commit to transfers after employer-to-employer (EE) separations and to transfers after employment-to-unemployment (EU) separations. The assumption that firms cannot make transfers to workers after EU separations is meant to capture that not all risk is insurable by firms, so that assets are used for precautionary savings. In the baseline model, assets are public information so firms and workers can contract on the saving decision of workers and firms know the assets of the workers they match with. I argue later that relaxing this assumption makes little difference.

I first show that allowing workers to trade assets influences optimal contracts, even when assets are public information, precisely because workers and firms cannot commit to transfers post EE or EU separations. Specifically, if firms and workers could commit to transfers post EE or EU separations, then the optimal contract can be implemented with zero asset holdings for workers. This means that the assumption that workers are hand-to-mouth is not restrictive in this case. Besides, I show that the optimal contract can use assets as a substitute for either type of transfers. For example, instead of implementing a transfer from workers to firms after EE separations, the contract can be implemented by reducing the worker's assets holdings and backloading wages. This ensures that the continuation value of workers falls after EE separations. Conversely, instead of implementing a transfer from firms to workers after EU separations, the contract can be implemented by increasing the worker's assets and frontloading wages. This ensures that the worker is insured against unemployment risk. In the quantitative model, neither type of transfers can be implemented so assets are used to substitute for both, leading to a trade-off where assets are used to enhance worker retention and to insure workers against unemployment risk[1].

This trade-off between worker retention and insurance can be characterized using the optimality conditions of the optimal contract. The first is a standard consumption growth condition, which also arises in optimal wage contracts with hand-to-mouth workers (e.g. Balke and Lamadon, 2022). The second is a new pseudo Euler equation, which highlights the different roles that assets play in optimal wage contracts. This equation shows that the optimal contract will use assets to smooth the worker consumption over time and across states, for a given path of wages. For instance, the contract will deplete the worker assets when wages are backloaded to smooth consumption over time. By contrast, the contract will increase the worker assets when workers face unemployment risk, to increase the precautionary savings of workers. The pseudo Euler equation also shows that firms manipulate assets to influence the search decision of workers through wealth effects as in Acemoglu and Shimer (1999). However, I show in the quantitative model that this effect is minuscule and thus does not influence optimal contracts much.

Consider how the presence of assets changes the optimal degree of wage backloading in optimal wage contracts, and thus the average wage growth. In designing contracts, firms face a trade-off between retaining and attracting workers. Remember that firms cannot control the worker search decision because it is private information. As a result, the optimal retention strategy for firms is to backload wages to make workers search for jobs with higher wages and lower job finding rates as in Burdett and Coles (2003) and Shi (2009). Firms know that in response to backloaded wages, workers will smooth

[1] These results clarify why assets play a role in my model whereas they do not in the literature on optimal unemployment insurance (e.g. Hopenhayn and Nicolini, 1997). In this literature, it is standard to assume that the government (the principal) can tax the worker (the agent) when she finds a job. This assumption is equivalent to assuming that transfers post EE separations can be implemented. There is no exogenous separation between the government and the worker so transfers for EU separations play no role. Hence the focus of this literature on hidden saving.

their consumption over time by consuming their existing assets. Therefore, relative to a model without assets firms can potentially backload wages significantly more. However, firms also know that workers face borrowing constraints so they might not be able to smooth consumption over time. Besides, they know that workers would like to save to self-insure against unemployment risk. In particular, backloading wages too much and making workers consume their savings at the beginning of a match makes contracts very unattractive to risk-averse workers who anticipate that their consumption will fall significantly if an unemployment shock occurs. Indeed, if one firm adopted a strategy of extreme wage backloading, it would have to offer much higher average wages to make its offer attractive relative to an offer that offers more stable wages and, hence, more stable consumption. The optimal contract thus balances the desire of firms to retain workers through backloaded wages, and the desire of workers to smooth consumption and self-insure against unemployment risk through frontloaded wages. Relative to a model with hand-to-mouth workers, this model therefore generates wage paths that can be more or less backloaded, and even frontloaded, and that depend on the worker's existing assets and ability to borrow.

Introducing assets into the model also influences how firms choose to pass productivity shocks through to workers and thus the volatility of wages. First, the pass-through to wages is larger when workers have access to financial markets than when they are hand-to-mouth. Second, and perhaps more surprisingly, firms also take advantage of workers' access to financial markets to influence the shape of the pass-through, and not just its size. To understand how, consider how firms respond to a positive persistent shock to productivity. As in model with hand-to-mouth workers, firms respond by increasing the wage of workers to reduce the quit rate. In contrast however, firms now respond with promises of higher wages in the future, thus backloading wage increases. In fact, if shocks are sufficiently persistent, the optimal response to a positive shock is to cut wages on impact and later increase wages significantly. Meanwhile, consumption increases smoothly over time as workers initially deplete their assets in anticipation of higher future income. Why is it optimal for firms to cut wages and deplete the worker assets on impact after a positive shock? When productivity increases, the profits of firms rise so the worker retention motive described before becomes stronger relative to the insurance motive so firms choose to backload wages more. They backload wages so much that wages can actually fall on impact.

I estimate the model using French administrative data to quantify how much insurance workers across the wealth distribution receive inside and outside the firm. The model is consistent with empirical evidence that wages grow on average with tenure and with evidence that both the pass-through of productivity shocks to wages and the marginal propensity to consume are positive but less than one. The model is also consistent with existing empirical evidence showing that the wage of asset-rich workers grows more on average and responds more to firm-level productivity shocks relative to asset-poor workers. Relative to a model with hand-to-mouth workers, I find that workers receive less insurance from firms in that their wages are more backloaded and the pass-through of productivity shocks to wages is higher. In this sense, the insurance that workers receive outside the firm crowds out the insurance that they receive inside the firm. Workers however receive more insurance overall relative to hand-to-mouth workers because their consumption is more stable over time and responds less to productivity shocks.

The model implies that wage growth is heterogeneous over the wealth distribution. Specifically, workers who start their career with relatively more assets receive wages that are more backloaded because they can smooth their consumption themselves. As a result, they experience higher wage growth but lower consumption growth than workers who start their career with relatively less assets. Wealthy workers also match with more productive firms and receive higher average wages because firms invest in better technology when they can backload wages more and worker mobility rates are low. Quantitatively, I find that the wages of workers at the top of the wealth distribution grows by 0.5% more on average per year and is 1% higher relative to workers at the bottom of the wealth distribution. This mechanism is consistent with evidence that workers with better access to financial markets receive wages that are more backlaoded (Guiso et al., 2012) and that workers with wealthy parents experience higher wages growth during their career than workers with poor parents (Halvorsen et al., 2022).

The model also implies that the pass-through of productivity shocks is heterogeneous over the wealth distribution. Specifically, the pass-through to wages is about twice larger for workers at the top of the wealth distribution relative to workers at the bottom of the distribution. The pass-through to consumption however is 3 times lower for workers at the top relative to workers at the bottom because these workers have a much lower MPC. This shows that wealthy workers receive less insurance from firms but more insurance overall against shocks. The reason why firms do not provide more insurance to poor workers is that these workers are at the bottom of the job ladder. As a result, their EE rate is high so employers are willing to let their consumption fluctuates more to optimize worker retention relative to workers at the top of the job ladder whose EE rate is much less sensitive to their consumption. This mechanism is consistent with empirical evidence from Fagereng et al. (2018) who show that the pass-through of firm-level productivity shocks to wages is increasing in wealth.

Taken together, these results have new implications for public policies because they show that the wealth of workers influence how much insurance workers receive against shocks and where this insurance is coming from. In particular, policies that relax the borrowing constraint of workers improve their ability to self-insure, just like wealthy workers. As a result, workers receive less insurance from firms in that their wages become more backloaded and more exposed to firm productivity shocks but they receive more insurance overall in that their consumption becomes less backloaded and respond less to shocks. Quantitatively, I find that letting workers borrow up to 2 quarters of income increases average wage growth 4 times but reduces average consumption growth 3 times, and it increases the pass-through to wages by 40% but reduces the pass-through to consumption by 30% for workers with less than 10 years of experience who are most affected by the policy. This policy also improves allocative efficiency as workers match with firms that are 1.5% more productive on average and receive average wages that are 0.6% higher.

Throughout, I have assumed that assets are public information. This implies that firms know how much assets workers have when they first match, and that the saving decision of workers is contractible. It is natural to wonder whether this assumption, which is somewhat unrealistic, has important implications for the optimal contract. I show that with CARA utility, the allocation is identical when assets are private or public information. This result implies that we can reinterpret the model as one where firms design wage contracts that workers select depending on their assets, and where workers choose how much to consume and save independently of firms. With CRRA utility however, the optimal contract designed under the assumption that assets are public information is no longer incentive compatible because of wealth effects on search. These deviations however appear to be quantitatively small, suggesting that the optimal contract would not be very different if they were taking into account these deviations. Those results are closely related, thought not identical, to those of Werning (2002) and Abraham and Pavoni (2008) on optimal unemployment insurance, and of Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023) on fixed-wage contracts with assets and directed search.

In this paper I focus on non-contingent assets, as opposed to more general securities, because in the data most assets held by households, such as cash, are non-contingent. This assumption has two important implications relative to the model where workers have access to complete financial markets studied by Stevens (2004). First, with complete markets the optimal contract no longer suffers from moral hazard. In particular, the worker buys the job from the firm by paying an upfront fee, and then receives a wage equal to the value of output. By contrast, with non-contingent assets, such a contract is not optimal because firms want to insure workers against the risk of not finding another job. As a result, the optimal contract is still subject to moral hazard[2]. Second, the model with complete markets makes predictions that are sharply at odds with the data. In particular, in my model the tenure profile of wages is consistent with the data precisely because workers face uninsurable unemployment risk, and there is limited pass-through of productivity shocks to wages, as in the data, only with incomplete markets.

This paper contributes to a recent literature bringing together labor market transitions and asset accumulation. An early example is Krusell, Mukoyama and Şahin (2010) who study precautionary savings in a DMP model where wages are set by Nash bargaining. Several articles have since introduced assets in search models with EE separations (Lise, 2013, Chaumont and Shi, 2022, Alves, 2022, Kaas, Lalé and Nawid, 2023, Caratelli, 2024) but in these models wage contracts are subject to ad hoc restrictions. Specifically, wages are assumed to be constant during matches, or assumed to change only when workers receive an outside offer. Instead, my paper is the first to study the determinants of worker mobility and assets in a model with optimal wage contracts. The main advantage of my approach is that it allows to study the sources of insurance that workers receive against labor market risk. Besides, I also show that fixed-wage contracts are inefficient in my environment because they abstract from the firm's desire to retain workers and the worker's demand for precautionary savings. Quantitatively, I find the gains from optimal contracts relative to fixed-wage contracts for firms to be quite large.

The paper starts in section 2 by presenting a new model with wage contracts and assets, which I characterize in section 3. Section 4 brings the model to data and quantifies determinants of income inequality and the amount of insurance that workers receive inside and outside the firm. Finally, section 5 revisits the assumption that assets are public information. Proofs are in the appendix.

2 A model with wage contracts and assets

I first present a new model with search frictions, dynamic wage contracts and assets. The model combines the optimal contract with search frictions of Menzio and Shi (2010) and the precautionary savings model of Bewley (1977).

[2]This is similar to what Shimer and Werning (2008) call the need to "insure against uncertain spell duration" in the unemployment insurance literature.

2.1 Environment

Time is discrete and runs forever.

Agents A continuum of ex-ante homogeneous workers can be employed or unemployed. Workers receive wage w when employed, and home production b when unemployed. They have period utility $u(c)$ over consumption and discount the future at rate β.

Firm are owned by foreign diversified investors, so they are effectively risk-neutral with discount rate r. An active firm is one that is matched with a single worker. The output from that match x_t follows the mean reverting process

$$x_t = (1-\rho)x_0 + \rho x_{t-1} + \sigma \nu_t$$

where ν_t are i.i.d. innovations with standard normal distribution, and ρ parametrizes the persistence of productivity. The mean productivity x_0 is selected by firms before they match with workers, and should be interpreted as the result of firm investment in worker training and in production technology. This component stays constant over time and lasts for the length of the match.

Financial markets This is a small open economy with foreign interest rate r. Workers can save using risk-free bonds a_{t+1} subject to a borrowing constraint, so that

$$a_{t+1} \geq 0$$

Timing Each period, the sequence of events is as follows

a) Productivity shocks ν_t and exogenous separations into unemployment occur

b) Employed workers and workers who were unemployed at the start of the period search for jobs; firms post vacancies; new matches are formed

c) Firms produce and pay current wages; workers make saving decision and consume

Directed search with assets There is a continuum of labor markets indexed by the promised value to a worker denoted v and the assets of workers a. Indexing labor markets by the worker promised value v is critical so that workers know where to search for a job. Indexing labor markets by the worker assets a is critical so that firms know how much profit they will generate from a match. Every period, workers choose in which labor market to search and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets.

I assume throughout that firms commit to delivering value v in markets indexed by v. I also assume in the baseline model that workers with assets a commit to searching only in markets indexed by a, and not elsewhere. In section 5, I will consider the possibility that workers with assets a search in markets indexed by $\tilde{a}$ so that firms might not know how much assets the workers they match with actually have.

Denote $\phi_u(v,a)$ and $\phi_e(v,a)$ the mass of unemployed and employed workers searching for a job and denote $\phi_f(v,a)$ the mass of vacancies posted by firms. Let κ denote the search intensity of employed workers relative to unemployed workers. In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u+\kappa\phi_e,\phi_f)$ turns workers searching for a job and vacancies into matches. Define the job finding rate $\tilde{\lambda}_w(\phi_u+\kappa\phi_e,\phi_f)$ as the probability that an unemployed worker finds a job, and the vacancy filling rate $\tilde{\lambda}_f(\phi_u+\kappa\phi_e,\phi_f)$ as the probability that a vacancy finds a worker. These probabilities are defined in the usual way as

$$\tilde{\lambda}_w(\phi_u+\kappa\phi_e,\phi_f)\equiv\frac{\mathcal{M}(\phi_u+\kappa\phi_e,\phi_f)}{\phi_u+\kappa\phi_e},\quad \tilde{\lambda}_f(\phi_u+\kappa\phi_e,\phi_f)\equiv\frac{\mathcal{M}(\phi_u+\kappa\phi_e,\phi_f)}{\phi_f}$$

Since these matching probabilities will depend on v and a in equilibrium, we can write them in short-hand notation as

$$\lambda_w(v,a)\equiv\tilde{\lambda}_w(\phi_u(v,a)+\kappa\phi_e(v,a),\phi_f(v,a)),\quad \lambda_f(v,a)\equiv\tilde{\lambda}_f(\phi_u(v,a)+\kappa\phi_e(v,a),\phi_f(v,a))$$

Unemployed workers Unemployed workers face a standard consumption-savings decision problem similar to Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023). They receive an endowment b, choose how much to save and consume and in which labor market v to search. Given the job finding probability, $\lambda_w(v,a)$, the value of unemployed workers satisfies

$$\begin{aligned}
U(a_t)=\max_{c_t,a_{t+1},v_{t+1}}\quad & u(c_t)+\beta\left[\lambda_w(v_{t+1},a_{t+1})v_{t+1}+(1-\lambda_w(v_{t+1},a_{t+1}))U(a_{t+1})\right]\\
\text{s.t.}\quad & c_t\leq(1+r)a_t+b-a_{t+1}\\
& a_{t+1}\geq 0
\end{aligned}$$

Appendix A.4 shows that the choice of savings a_{t+1} follows a standard Euler equation where workers smooth their consumption by depleting their savings over time. The choice of search v_{t+1} follows a standard trade-off: searching in a high-v labor market brings a higher value v conditional on a match, but it will turn out that these matches occur with lower probability because $\lambda_w(v, a)$ will decrease with the value v in equilibrium[3].

Employed workers Employed workers find a new job in market v when their current asset is $a_t$with probability $\kappa\lambda_w(v, a_t)$. Existing matches break up and workers separate into unemployment with exogenous probability δ[4].

Contracts Optimal wage contracts specify wages w_t and transfers after separations into employment $(1 + r)\tau_t^{ee}$ or unemployment $(1 + r)\tau_t^{eu}$ for each history of shocks and conditional on the worker's initial asset position a_0 and on delivering initial value v to workers. Productivity is public information and I assume that it is unfeasible for firms to make counteroffers to their workers when they receive outside job offers[5]. In the baseline model, I assume that the savings decision of workers a_{t+1} is public information so it is contractible but I relax this assumption in section 5.

The contract is subject to two sets of contracting frictions, which both play a critical role in the analysis. First, the worker search decision is private information, which leads to moral hazard. Second, workers and firms cannot commit to transfers post separations into employment τ_t^{ee} or unemployment τ_t^{eu}. This assumption implies that these transfers must be equal to 0 after any history in the optimal contract.

I show in section 3.1 that letting workers trade risk-free bonds influences the optimal allocation precisely because these transfers post EE and EU separations cannot be implemented. The assumption that transfers post EE separations cannot be implemented is standard in the literature on optimal contract and is usually justified on the ground that bonded labor is prohibited by law (e.g. Stevens, 2004). The assumption that transfers post EU separations cannot be implemented is more controversial because firms do make severance payments in practice. However, even formal commitment about severance pay

[3]In models without assets and where x_0 is homogeneous across firms, the search policy exists and is unique (see Menzio and Shi, 2010) because the job finding rate $\lambda_w(v)$ is concave in equilibrium. In my model however, the search and savings policies might not be unique because the equilibrium job finding rate $\lambda_w(v, a)$ is not always concave as firms select their average productivity x_0 and because there might be complementarity between the search and savings decision of workers. I solve the model using the optimality conditions of the worker and verify ex-post that the solution corresponds to the unique solution numerically.

[4]To keep the notation simple, I do not allow for quits into unemployment. However, I verify ex-post that this restriction does not bind in the quantitative model. I find that because the endowment value b is well below firm productivity, almost no worker would be better off quitting in model-simulated data.

[5]This assumption can be formally justified as follows: counteroffers are private information to workers, and expire before workers can return to their current employers to negotiate higher wages. These assumptions ensure that it is optimal for current employers not to respond to outside offers.

can be subject to interpretation[6]. Besides, firms might not able to implement severance payments to workers if the separation occurs because of firm bankruptcy. Beyond this specific assumption, what is critical for the analysis is that workers face some risk that firms cannot insure directly and therefore require insurance outside firms in the form of precautionary savings[7].

2.2 Optimal contracts

Following previous work on dynamic contracts, I write the contract recursively in terms of promised values and continuation values instead of histories of shocks[8]. Denote V_t the promised value of an employed worker at the start of the period. The state of a match at the beginning of the period is the worker promised value V_t, the asset of the worker a_t, the firm average productivity x_0 and current productivity x_t. Denote by $s_t \equiv V_t, a_t, x_0, x_t$ the vector of current state variables.

The components of the contract at time t are the wage paid today, the transfers post EE and EU separations, the savings decision of workers and a set of continuation values for each state tomorrow. Formally, these components are represented by the functions

$$w_t\left(s_t\right), \quad \tau_t^{ee}\left(s_t\right), \quad \tau_t^{eu}\left(s_t\right), \quad a_{t+1}\left(s_t\right), \quad V_{t+1}(s_t, x_{t+1})$$

A *contract* is a collection of these functions for all t. In the recursive formulation below, we write the components of the contract as $w_t, \tau_t^{eu}, \tau_t^{ee}, a_{t+1}, V_{t+1}(x_{t+1})$ without explicitly mentioning state variables. It will also be convenient to define the continuation value of workers at the current job as

$$W_t \equiv u(c_t) + \beta \mathbb{E}_{x_{t+1}}\left[V_{t+1}(x_{t+1}) | x_0, x_t\right] \tag{1}$$

where $c_t + a_{t+1} = (1+r)a_t + w_t$.

Worker value Given the contract, the worker chooses a search strategy to maximize the present value of utility. The value of a worker satisfies

$$V_t = \delta U(a_t + \tau_t^{eu}) + (1-\delta) \max_{v_t} \left[\kappa \lambda_w(v_t, a_t + \tau_t^{ee}) v_t + (1 - \kappa \lambda_w(v_t, a_t + \tau_t^{ee}))\, W_t\right] \tag{2}$$

[6] For example, severance pay only applies if termination occurs for reasons outside of worker's control.

[7] There are other ways to generate such need for insurance outside the firm. For instance, if workers receive income shocks that are private information to workers (e.g. health expense shocks, spousal income shocks), then the amount of insurance that firms can provide is limited (as in Cole and Kocherlakota, 2001).

[8] I abstract from randomized contracts to keep notation simple.

In equation (2), the first term is the continuation value of a worker who becomes unemployed at time t with assets a_t and receive transfer from firms τ_t^{eu}. The second term depends on the probability that a worker finds another job $\kappa\lambda_w(v_t, a_t + \tau_t^{ee})$ and on the value that the worker receives if an EE separation occurs v_t. A worker with asset a_t who finds a new job will start that job with asset $a_t + \tau_t^{ee}$. A worker who does not find a new job receives the continuation value W_t.

Equation (2) shows how the assumptions of hidden search and limited commitment about transfers interact. Notice that moral hazard arises because the worker's search decision v_t only depends on the surplus that workers get from EE separations $v_t - W_t$, and not on the firm value. Thus, relative to the search policy that the worker chooses, a firm with a positive value would prefer the worker to search instead in markets with a higher value v_t and a lower job finding rate $\lambda_w(v_t, a_t + \tau_t^{ee})$ because it wants to retain the worker. Since the worker search decision v_t is private information, the firm cannot control it directly and instead influences the worker's decision indirectly by manipulating the continuation value at the current job W_t and the job finding rate $\lambda_w(v_t, a_t + \tau_t^{ee})$. How can the firm manipulate the job finding rate? If firms and workers could commit to transfers, the firm could enforce a transfer τ_t^{ee} from the worker after EE separations. This would reduce the worker's asset at the next job, thus lowering the job finding rate in equilibrium. When these transfers cannot be implemented, the firm can instead manipulate the worker's assets a_t. However, assets also influence the continuation value of workers who become unemployed $U(a_t + \tau_t^{eu})$ so firms optimally manipulate the workers' assets to influence their EE mobility only to some extent. This dual role for assets generates a trade-off that I describe in details in section 3.1.

Optimal contracts Denote the optimal search policy $v(W_t, a_t + \tau_t^{ee})$[9], the implied EE probability as

$$p_t \equiv p(W_t, a_t + \tau_t^{ee}) \equiv \kappa\lambda_w(v(W_t, a_t + \tau_t^{ee}), a_t + \tau_t^{ee})$$

and the worker expected surplus from EE separations as

$$S_t \equiv S(W_t, a_t + \tau_t^{ee}) \equiv \kappa\lambda_w(v(W_t, a_t + \tau_t^{ee}), a_t + \tau_t^{ee}) \left(v(W_t, a_t + \tau_t^{ee}) - W_t\right)$$

Finally, denote $\Pi(s_t)$ the present value of profits for a firm matched with a worker who has promised value V_t and assets a_t when average productivity is x_0 and currently x_t. Taking as given the value of unemployment $U(a)$, the equilibirum job finding rate

[9] As for unemployed workers, the search decision might not be unique in this environment. I check it ex-post numerically when I solve the model.

$\lambda_w(v,a)$, and the search policy of workers $v(W,a)$, the optimal contract solves

$$\begin{aligned}\Pi(s_t) = \max_{w_t,\tau_t^{eu},\tau_t^{ee} c_t,a_{t+1},V(x_{t+1})} (1-\delta)(1-p(W_t,a_t+\tau_t^{ee}))\left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}[\Pi(s_{t+1})|x_0,x_t]}{1+r}\right) \\ -\delta(1+r)\tau_t^{eu} - (1-\delta)p(W_t,a_t+\tau_t^{ee})(1+r)\tau_t^{ee}\end{aligned} \tag{3}$$

subject to

$$\begin{aligned}
&\text{(PK)}: && V_t \leq \delta U(a_t+\tau_t^{eu}) + (1-\delta)\left[W_t + S(W_t,a_t+\tau_t^{ee})\right]\\
&\text{(Budget)}: && c_t + a_{t+1} = (1+r)a_t + w_t\\
&\text{(BC)}: && a_{t+1} \geq 0\\
&\text{(LC)}: && \tau_t^{eu},\tau_t^{ee} = 0
\end{aligned}$$

where $W_t = u(c_t) + \beta\mathbb{E}_{x_{t+1}}\left[V_{t+1}(x_{t+1})|x_0,x_t\right]$ and $s_{t+1} \equiv V(x_{t+1}), a_{t+1}, x_0, x_{t+1}$.

The contract maximizes the present value of profits, where $(1-\delta)(1-p(W_t,a_t+\tau_t^{ee}))$ is the probability that the worker remains within the current match this period. The first constraint (PK) is the promise keeping constraint, stating that the value the worker gets from the contract either at the current job, through unemployment or at future jobs must deliver at least the promised value V_t. The second constraint (Budget) is the budget constraint of the worker, and the third constraint (BC) is the borrowing constraint. The last constraint (LC) is the worker and firm limited commitment constraint. We wrote the optimal contract taking as given the optimal search policy of workers $v(W,a)$, so the incentive compatibility constraint for search is implicit in the definition of $p(W,a)$ and $S(W,a)$.

Value of new matches In the first period of employment, firms also solve (3) except that there is no EU separations ($\delta = 0$), no EE separations ($p_t = S_t = 0$) and no productivity shock ($\nu_t = 0$). Denote the firm value in the first period by $\Pi_0(V_t,a_t,x_0)$.

2.3 Equilibrium

Free entry Firms post vacancies in each labor markets subject to a free entry condition. Firms choose in which market (v,a) to post vacancies and which technology x_0 to adopt. The unit cost of posting a vacancy with technology x_0 is $k(x_0)$ with $k,k',k'' > 0$. The free entry condition is

$$\max_{x_0} \; -k(x_0) + \lambda_f(v,a)\Pi_0(v,a,x_0) \leq 0 \tag{4}$$

with equality for each active market (v,a).

Making the choice of x_0 endogenous is a standard way to generate fixed heterogeneity in productivity across firms (Acemoglu and Shimer, 2000), which is widely accepted as one of the main drivers of worker mobility. In equilibrium, firms that post vacancies in markets with higher values v or lower assets a will select higher productivity. A convenient implication of this formulation is that it increases the upper bound on the worker value v for active markets. This means that even workers with relatively high wages can search in markets with higher values, where firms are very productive and where the job finding rate is very low. As a result, even these workers can switch jobs, which is critical to generate some of the quantitative results from section 4.3. By contrast, when x_0 is fixed across firms, workers with relatively high wages arrive quickly at the top of the job ladder where their EE separation rate is 0[10]. The assumption that firm initial productivity x_0 is endogenous also turns out to interact with the optimal contract and the worker's initial asset holding, as I explain in section 4.2.

Definition of an equilibrium An equilibrium is a set of value functions, policies and matching rates for each labor market (v, a) such that i) the unemployed worker policies maximizes the unemployment value, ii) the firm and employed worker policies satisfy the optimal contract, iii) the free entry condition is satisfied and iv) the job finding and vacancy filling rates are consistent with the matching function. The laws of motion for the distributions $D^u(a)$ and $D^e(s)$, defined as usual, are satisfied given the policies.

3 The role of assets in wage contracts

I now characterize the optimal wage contract by describing how wages, assets and consumption change with tenure and in response to productivity shocks. I will emphasize how assets alter contracts relative to models with optimal wage contracts and hand-to-mouth workers (as in Burdett and Coles, 2003) and relative to models with hidden search from the unemployment insurance literature where assets play no role (as in Hopenhayn and Nicolini, 1997). I first clarify that assets influence optimal contracts because they substitute for missing transfers post EE and EU separations. Because firms cannot implement those transfers, they decide to use workers' assets to optimize worker retention further but also to insure them against unemployment risk. I then use the optimality conditions of the optimal contract, including a new pseudo Euler equation that I derive, to show how the paths of wages, assets and consumption depend on this new trade-off between

[10] Alternative ways to make the job ladder longer include making the EU separation probability δ endogenous as in Balke and Lamadon (2022) or assuming that workers receive iid preference shocks for jobs that are private information as in Souchier (2023).

worker retention and insurance. Finally, I describe how firms pass productivity shocks through to workers in this model. In this section, I work under the assumption that assets are publicly observable but I discuss this assumption in more length in section 5.

3.1 Assets substitute for missing transfers

This section clarifies that assets influence optimal contracts because they substitute for missing transfers post EE and EU separations. I state the main result in proposition 1 and then explain it using figure 1 for illustration.

Proposition 1. *Consider the following 3 cases regarding transfers τ_t^{ee} and τ_t^{eu}:*

1. Assume that firms and workers can commit to both τ_t^{ee} and τ_t^{eu}, that is the optimal contract (3) does not need to satisfy constraint (LC). Then, the path of consumption and EE probability are identical in optimal contract with assets (3) and in a restricted contract with hand-to-mouth workers, that is with the additional constraint $a_{t+1} = 0$. Denote the optimal paths of consumption and EE probability as c_t^*, p_t^* and denote the paths of transfers that implement the optimal contract with $a_{t+1} = 0$ as $(\tau_t^{ee})^*$ and $(\tau_t^{eu})^*$.

2. Assume instead that firms and workers can commit to τ_t^{eu} but not to τ_t^{ee}, that is constraint (LC) in the optimal contract (3) is replaced by $\tau_t^{ee} = 0$. Assume further that productivity is constant within matches and that workers face no borrowing constraint. Then, the paths of consumption and EE probability are identical to case 1, that is $c_t = c_t^*$ and $p_t = p_t^*$ for all t. Besides, the optimal path for assets satisfies

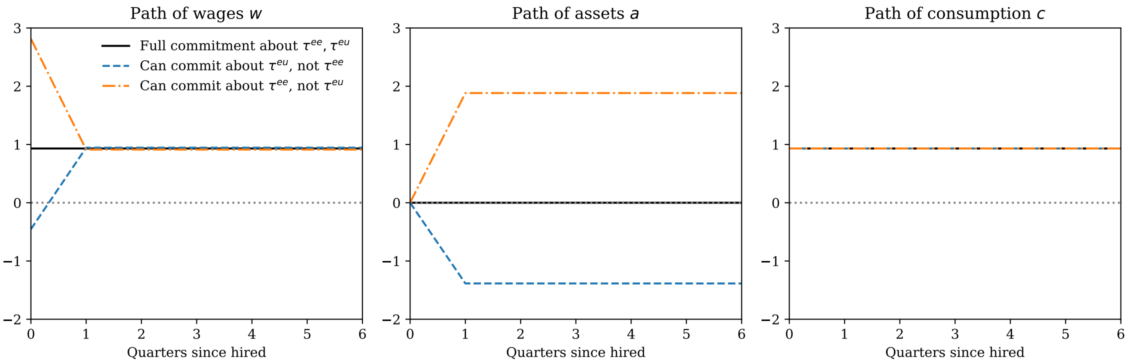
$$a_{t+1} = (\tau_{t+1}^{ee})^* \quad \forall t$$

3. Assume instead that firms and workers can commit to τ_t^{ee} but not to τ_t^{eu}, that is constraint (LC) in the optimal contract (3) is replaced by $\tau_t^{eu} = 0$. Assume further that productivity is constant within matches and that workers face no borrowing constraint. Then, the paths of consumption and EE probability are identical to case 1, that is $c_t = c_t^*$ and $p_t = p_t^*$ for all t. Besides, the optimal path for assets satisfies

$$a_{t+1} = (\tau_{t+1}^{eu})^* \quad \forall t$$

Proof. See appendix A.1. □

Figure 1: Assets substitute for missing transfers post EE and EU separations



Note: paths of wages, assets and consumption for an employed worker hired from unemployment at $t = 0$ with zero initial asset. The solid black line shows the model where both transfers τ^{ee} and τ^{eu} can be implemented, and where it is assumed that $a_{t+1} = 0$. The dashed blue line shows the model where τ^{eu} but not τ^{ee}. The dash-dotted orange line shows the model where τ^{ee} but not τ^{eu}. In all models, productivity is assumed to be constant across firms and over time ($x_t = 1$) and workers and firms have the same discount factor $\beta(1+r) = 1$.

Case 1 from proposition 1 shows that the optimal allocations c_t and p_t are identical whether workers are hand-to-mouth or can trade risk-free bonds, provided that firms and workers can commit to transfers τ_t^{ee} and τ_t^{eu}. In this case, letting workers trade risk-free bonds is therefore irrelevant when these trades are public information. This is a well known result in the optimal unemployment insurance literature: the principal can implement the optimal contract in many different ways, including by saving on behalf of the agent and setting $a_{t+1} = 0$ for the duration of the match (e.g. see Werning, 2002). Critically, this result requires that transfers τ_t^{ee} and τ_t^{eu} can be chosen freely in the optimal contract. This assumption is in fact standard in the literature on optimal unemployment insurance[11], but not in the literature on optimal wage contracts[12]. This is the reason why letting workers trade risk-free bonds influences optimal wage contracts even when they are public information whereas they do not influence optimal unemployment insurance contracts. The solid black line from figure 1 shows one implementation of the optimal contract, the one satisfying $a_{t+1} = 0$ for all t. In this specific example, productivity is

[11]Since Hopenhayn and Nicolini (1997), it is common practice in this literature to assume that the government (the principal) can tax workers (the agent) when they find a job. This assumption is equivalent to assuming that workers can commit to transfers when they find a job, that is τ_t^{ee} can be freely chosen. Furthermore, in these models there is no exogenous separation between the government and unemployed workers ($\delta = 0$) so τ_t^{eu} is irrelevant. Interestingly, the seminal article of Shavell and Weiss (1979) did not assume that the government could tax workers but the literature on hidden savings has taken the model of Hopenhayn and Nicolini (1997) as a starting point instead.

[12]The literature on optimal wage contracts (e.g. Burdett and Coles, 2003; Shi, 2009) assumes that workers cannot commit to transfers post EE separations, τ_t^{ee}, not only because it is realistic but also because it is critical to generate wages that are backloaded, which is a key prediction of these models. If workers could commit to τ_t^{ee}, then wages would be either constant or frontloaded.

constant across firms and over time, the worker is matched from unemployment with zero initial asset and $\beta(1+r) = 1$. The optimal contract implements a constant path for consumption with a *constant wage*.

Case 2 from proposition 1 shows that the optimal allocations c_t and p_t remain the same as in case 1 even if firms and workers can only commit to transfers post EU separations τ_t^{eu} but not to transfers post EE separations τ_t^{ee}, provided that workers can trade risk-free bonds. In this sense, the optimal contract uses assets as a substitute for transfers τ_t^{ee} to influence worker mobility decisions. The dashed blue line from figure 1 shows how the optimal contract is implemented in this case. Wages are now *backlaoded*, and in fact negative during the first period of employment. The optimal contract makes the worker borrow and consumption is constant over time, as in case 1. Therefore, this figure confirms that the optimal allocation is identical in cases 1 and 2 even though the implementations of the contract differ.

To understand why assets can be used to substitute for transfers post EE separations τ_t^{ee}, it is useful to consider how the optimal contract uses these transfers in the first place. The main contracting friction in the optimal contract is that the search decision of workers is private information. This means that the worker's search decision, and the implied EE separation rate, are chosen to maximize the worker's value, not the joint value from the match. Often, in the context of optimal wage contracts, this means that workers are too likely to switch jobs so firms want to reduce worker mobility. When firms and workers can commit to transfers, the optimal contract thus requires workers to pay a fee back to their previous employer when they switch jobs, that is $\tau_t^{ee} < 0$. This reduces the worker's continuation value at the next job and ensures that the worker's search decision maximizes the joint value. When these transfers cannot be implemented, the optimal contract instead backload wages to reduce the worker's assets as in figure 1. This too reduces the worker's continuation value at the next job and ensures that the worker's search decision maximizes the joint value[13].

Case 3 from proposition 1 considers the opposite assumptions about transfers. It shows that the optimal allocations c_t and p_t remain the same as in case 1 even if firms and

[13]In this specific example, introducing assets solves the model hazard because productivity is constant across firms. With heterogeneity in productivity, wages and consumption would not be constant over time even with transfers. This is because with constant wage the consumption of a worker matched with a low-productivity firm would jump up when she finds a better job. It is thus optimal for firms to insure the worker against this risk by increasing consumption at the current job, and reducing assets to reduce consumption at the future job. However, because the worker search decision is still private information, this brings back moral hazard. The optimal contract thus reduces the consumption and assets of workers over time, in a similar way as the government reduces unemployment benefits and increases taxes with unemployment duration in Hopenhayn and Nicolini (1997).

workers can only commit to transfers post EE separations τ_t^{ee} but not to transfers post EU separations τ_t^{eu}. Here, the optimal contract uses assets as a substitute for transfers τ_t^{eu} to insure workers against unemployment risk. The dash-dotted orange line from figure 1 shows how the optimal contract is implemented in this case. Wages are now *frontloaded*. The optimal contract makes the worker save and consumption is constant over time, as in case 1. As before, the allocations are the same as in case 1 but the implementations differ.

To understand why assets can be used to substitute for transfers post EU separations τ_t^{eu}, consider first how firms provide insurance to workers when these transfers are available. In this case, firms implement a transfer in the event of an exogenous separation into unemployment, effectively giving workers severance payment to insure them against unemployment risk. When these transfers cannot be implemented firms instead help workers self-insure against unemployment by making sure that workers have enough assets to smooth consumption themselves if they become unemployed. This is achieved by frontloading wages in the first period, which can be interpreted as paying workers a hiring bonus. In fact, the optimality conditions of the optimal contract show that firms achieve perfect insurance this way because $(1+r)\beta u'(c_{t+1}^u) = u'(c_t)$, meaning that the marginal utility of consumption remains constant after unemployment shocks.

Cases 2 and 3 also make the additional assumptions that workers should be able to borrow and that productivity is constant within matches. The reason for letting workers borrow is that transfers can be negative so replicating them implies to reduce the worker's assets. When the worker's initial asset is low, this may require workers to borrow. The reason for productivity to remain constant is that transfers are chosen after shocks are realized whereas assets a_{t+1} are non-contingent and chosen in the previous period. These conditions are really binding only for case 2, as figure 1 illustrates, because in case 3 the optimal contract usually implies a transfer from firms to workers and because τ_t^{ue} are independent of current productivity.

Taken together these results show that assets are used to substitute for missing transfers post EE and EU separations. When transfers post EE separations τ_t^{ee} cannot be implemented, firms use worker's assets to optimize worker retention. When transfers post EU separations τ_t^{eu} cannot be implemented, firms use worker's assets to help them self-insure against unemployment risk. In the model from section 2, neither τ_t^{ee} nor τ_t^{eu} can be implemented so firms face a trade-off when they choose how to use worker's assets in the optimal contract. The next section characterizes this trade-off using the optimal conditions of the contract (3).

3.2 Implications for tenure profiles

The previous section showed that letting workers trade risk free bonds leads to a trade-off between optimizing worker retention and helping worker self-insure against unemployment risk. This section uses the optimality conditions of the contract (3), including a new pseudo Euler equation, to characterize this trade-off and emphasize new implications relative to wage contracts with hand-to-mouth workers. In this section, we focus on tenure profiles that describe how wages, consumption and assets change with tenure at the firm even when productivity remains constant. In the next section, we will discuss how firms pass productivity shocks through to workers.

The consumption growth condition Our starting point is the following consumption growth condition

$$\frac{1}{u'(c_t)} - \frac{\beta(1+r)}{u'(c_{t-1})} = -\frac{p_W(W_t, a_t)}{1-p(W_t, a_t)} \left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0, x_t\right]}{1+r} \right) \tag{5}$$

which is derived in details in appendix A.2 from the optimality conditions of the contracting problem. This equation is similar to theorem 1 in Burdett and Coles (2003), lemma 3.2 in Shi (2009) and proposition 2 in Balke and Lamadon (2022), except that consumption c_t now replaces wages w_t on the left-hand side.

The intuition behind equation (5) is well understood in the literature. It equates the benefits from backloading wages on the right with the costs on the left. Backloading wages means that firms reduce w_{t-1} to increase w_t, which increases the continuation value at the current job W_t. The benefit of backloading wages is to reduce the worker EE separation rate. This benefit depends on the extend to which promising workers higher values will reduce the EE rate, captured by the term $p_W(W_t, a_t)$, and on the value that firms get from the match, captured by the term in parenthesis.

Backloading wages is costly because it generates a gap in the marginal utility of consumption of workers over time. When workers are hand-to-mouth, backloading wages implies backloading consumption because $c_t = w_t$ so the cost of backloading is straightforward and equation (5) is enough to characterize the optimal contract, together with the definition of the worker and firm values $W_t, \Pi(s_t)$. By contrast, when workers can trade risk-free bonds, consumption and wages are not always equal so equation (5) must be combined with a new optimality condition relating wages, consumption and assets. This new condition is the pseudo Euler equation that we derive next.

The pseudo Euler equation This equation is derived from the optimality condition for assets in the optimal contract. The surprising result is that this condition takes the familiar form of an Euler equation, except for an additional term due to wealth effects on search. This is not obvious from inspection of the optimal contract (3) because the first order condition includes terms like $S_a(W_{t+1}, a_{t+1})$, which describes how the worker surplus from EE separations depends on assets. This term in turn depends on the fact that assets influence the search decision of workers $v(W_{t+1}, a_{t+1})$ and the matching rate in each market $\lambda_w(v_{t+1}, a_{t+1})$. We now manipulate this first order condition to show how it turns into the pseudo Euler equation from proposition 2, and then use this equation together with equation (5) to analyze how wages, consumption and assets change with tenure.

The first step to derive the pseudo Euler equation is to understand how assets influence job finding rates. For this, we turn to the free entry condition (4) that relates the value of new matches Π_0 and the vacancy filling rate λ_f (and thus the job finding rate λ_w through the matching function). First, notice that $\Pi_0(v, a, x_0) = \Pi_0(v, 0, x_0) + (1 + r)a$, so the value of new matches increases in the asset of the worker. Next, notice that the firm value Π_0 is strictly decreasing in the worker promised value v from the envelope condition. This suggests the existence of an indifference condition for firms between matching with a worker in a market with high value and high assets, and in a market with low value but also low assets. This also means that in equilibrium firms post relatively more vacancies in markets where workers ask for low values v, and in markets where they have high initial assets a. This relation is formally captured by the following equation, derived by combining the free entry condition with the envelope conditions,

$$\partial_a \lambda_w(v_{t+1}, a_{t+1}) = -\partial_v \lambda_w(v_{t+1}, a_{t+1})(1 + r)u'(c^{ee}_{t+1}) \tag{6}$$

where c^{ee}_{t+1} is the consumption of the worker at the next job after an EE separation. This equation states that, in equilibrium, increasing the assets of workers by 1% has the same effect on the EE probability than inducing workers to search in a market where the worker value is $(1 + r)u'(c^{EE}_{t+1})$% lower.

The next step is to combine this equation with the optimality condition for search v

$$\begin{aligned} S_a(W_{t+1}, a_{t+1}) &= \partial_a \lambda_w(v_{t+1}, a_{t+1}) \left[v_{t+1} - W_{t+1}\right] \\ &= -\partial_v \lambda_w(v_{t+1}, a_{t+1})(1 + r)u'(c^{ee}_{t+1}) \left[v_{t+1} - W_{t+1}\right] \\ &= p(W_{t+1}, a_{t+1})(1 + r)u'(c^{ee}_{t+1}) \end{aligned}$$

The first line uses the definition of $S(W, a)$ and the envelope theorem, the second line uses equation (6) and the third line uses the optimality condition for search v. Intuitively, raising the assets of workers increases their job finding rate so workers respond optimally by searching for jobs with a higher value, and thus a higher consumption at the next job. Thus, this equation shows that under the optimal contract increasing the worker's assets marginally by Δa has a similar effect on the worker value than increasing consumption at the next job by $(1+r)\Delta a$, which is what an Euler equation would imply. This is why the optimality condition for assets a_{t+1} in the optimal contract can ultimately be written as a pseudo Euler equation.

Proposition 2. *The optimal contract satisfies a pseudo Euler equation*

$$\frac{u'(c_t)}{\beta(1+r)} \geq \delta u(c^u_{t+1}) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p_{t+1}u'(c^{ee}_{t+1}) + (1-p_{t+1})u'(c_{t+1})|x_0, x_t\right] - \mathcal{W}_t \tag{7}$$

where wealth effects on search $\mathcal{W}_t$ are

$$\begin{aligned}\mathcal{W}_t \equiv \quad & (1-\delta)\frac{u'(c_t)}{\beta(1+r)}\mathbb{E}_{x_{t+1}}\Bigg[\left(u'(c_{t+1})p_W(W_{t+1}, a_{t+1}) + \frac{p_a(W_{t+1}, a_{t+1})}{1+r}\right) \\ & \times\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_0, x_t\Bigg]\end{aligned}$$

Equation (7) holds with equality when the borrowing constraint (BC) does not bind.

Proof. See appendix A.3. □

Equation (7) can be separated into two groups of terms. The first group includes everything but $\mathcal{W}_t$ and constitutes a standard Euler equation. It states that the marginal utility of consumption must be equalized over time and across states. Specifically, given a path for wages, the optimal contract will seek to use the worker's ability to access financial markets to smooth consumption over time within the match, but also across states after UE or EE separations. This ability to smooth the worker consumption using financial markets instead of wages is quantitatively the main difference between this model with assets and the model with hand-to-mouth workers so we will return to it shortly.

Before this, let us briefly describe the second group of terms gathered in $\mathcal{W}_t$. This term shows that firms use assets to influence the worker search decision through wealth effects. This mechanism here is identical to Acemoglu and Shimer (1999) who describe how unemployment benefits and assets influence unemployed workers' search decision in a directed search model. In a directed search model, searching for a job is similar to selecting a lottery with payoff v and winning probability $\lambda_w(v, a)$. Whether workers select a lottery with high risk and high payoff or a lottery with low risk and low payoff depends on their preferences. With CRRA utility, $u(c) = c^{1-\gamma}/(1-\gamma)$, workers with low assets are effectively more risk averse and thus select a safer lotteries in which jobs are easier to get but yield a lower value. How does this influence the contract? When the firm continuation value is positive, the firm wants to retain its worker and thus influences the search decision to reduce the EE rate. Therefore, relative to a model without wealth effects, the firm increases the asset of workers to make them select riskier lotteries, that is search in market with a higher value v and lower job finding rate $\lambda_w(v, a)$. In section 4, I show that wealth effects on search are quantitatively minuscule so they play a minor role in shaping optimal contracts. Furthermore, this effect is exactly 0 when utility is CARA, that is $u(c) = -\exp(-\gamma c)/\gamma$. For these reasons, I abstract from them when I describe the tenure profile of wages and consumption next.

Implications for tenure profiles We are now ready to combine equations (5) and (7) to analyze how assets influence the tenure profiles for wage and consumption.

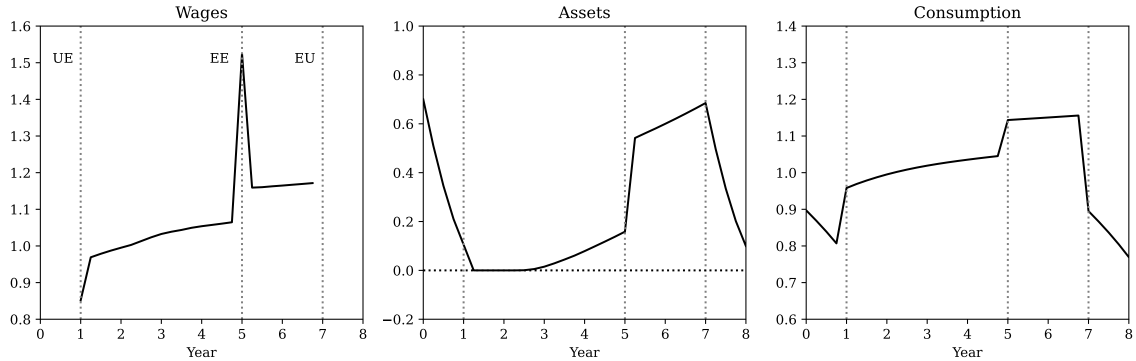
Consider first how firms set consumption and assets over time, given a path for wages. Assume for simplicity that wages are backloaded, as is generally the case in these models. If firms were to replicate the allocation with hand-to-mouth workers and set $c_t = w_t$, the consumption of workers would also be backloaded. This would make contracts relatively unattractive to workers with concave utility, so firms would need to pay workers high wages to attract workers relative to a firm offering contracts with wages that are less backloaded. Now consider how firms can improve on this allocation by setting $c_t \neq w_t$ using equation (7). On one hand, firms could deplete the worker's existing assets to smooth consumption over time. In equation (7), this means equalizing the terms $u'(c_t)/\beta(1+r)$ and the term $(1-\delta)\mathbb{E}_{x_{t+1}}\left[p_{t+1}u'(c^{ee}_{t+1}) + (1-p_{t+1})u'(c_{t+1})|x_0, x_t\right]$. In this case, backloading wages is less costly because assets are used to smooth consumption over time. On the other hand, firms could instead increase the worker's assets to improve insurance against unemployment risk. In equation (7), this means equalizing the terms $u'(c_t)/\beta(1+r)$ and the term $\delta u(c^u_{t+1})$. In this case, backloading wages is more costly because it makes it harder to increase precautionary savings.

Consider now how firms set wages over time, knowing that they can smooth consumption using the worker's assets. When the cost of backloading wages is low, firms know that they can smooth the worker's consumption using their existing assets despite wages being backloaded. As a result, they choose to backload wages even more to enhance worker retention. In the extreme, we recover case 2 from proposition 1 where assets are used to substitute for transfers post EE separations. By contrast, when the cost of backloading wages is high, firms choose to backload wages less to help them self-insure against unemployment risk. In the extreme, we recover case 3 from proposition 1 where assets are used to substitute for transfers post EU separations. Therefore, the degree of wage backloading depends on whether firms can use the worker's access to financial markets to smooth their consumption over time, or whether they want to help workers self insure against unemployment.

Relative to a model with hand-to-mouth workers, this model can generate wages that are much more backloaded, but also much less. In fact, wages can even be frontloaded so a new implication is that firms sometimes offer hiring bonuses to workers. Furthermore, the extend to which wages are backloaded depends on the amount of uninsurable risk that workers face, and on their ability to smooth consumption using their existing assets or through borrowing. In general, wealthy workers and workers who can borrow will receive wages that are more backloaded. In section 4, I show that these implications are consistent with empirical evidence and that they matter for public policies that relax the borrowing constraint of workers.

Life-cycle dynamics Figure 2 illustrates the implications of the optimal contract for the life-cycle dynamics of wages, assets and consumption. In this specific example, the worker starts unemployed at $t = 0$ with existing assets $a_0 = 0.7$. The worker finds a job at year 1, makes an EE separation at year 5 and an EU separation at year 7. The left panel shows that wages increase during the first job between years 1 and 5. The worker then receives a hiring bonus when switching jobs, and wages mildly increase later on during the second job between years 5 and 7. The middle panel shows that the worker depletes assets when unemployed and accumulates assets when employed. Finally, the right panel shows that consumption falls while the worker is unemployment and rises while the worker is employed. Consumption jumps during UE, EE and EU transitions and is smooth otherwise.

Figure 2: Life-cycle dynamics for wages, consumption and assets



Note: example of paths for wages, assets and consumption for a worker who is unemployed with $a_0 = 0.7$ at $t = 0$. The dotted vertical lines show when the worker experiences UE, EE and EU separations. The paths are computed in the quantitative model where productivity is heterogeneous across firms and shocks are expected to occur but where the realization of productivity shock ν_t happen to be null.

What accounts for the life-cycle dynamics in figure 2? Given the path of wages, the paths of assets and consumption satisfy the pseudo Euler equation (7), meaning that workers smooth consumption over time and state. The path for wages reveals how firms balance their desire to retain workers with their desire to insure them. Specifically, wages are backloaded during the first job. In fact, wages are so backloaded that the worker does not accumulate precautionary savings at all during approximately two years and consumes her entire savings during the first period of employment. If workers could borrow, firms would backload wages even more. The reason for this backloading of wages is that workers hired from unemployment are at the bottom of the job ladder. As a result, they receive a low wage so the firm value from the match is high. Besides, the EE separation rate of these workers is high so backloading their wages leads to a large reduction in their quit rate ($p_W(W, a)$ is large). This implies that the benefits of backloading wages on the right of equation (5) is large. Furthermore, the income of this worker does not fall too much if they become unemployed because the wage is low so the worker does not want to accumulate much precautionary savings. This means that the cost of backloading wages on the left of equation (5) is small. These observations imply that the retention motive is stronger than the precautionary savings motive so firms choose to backload wages significantly. The backloading of wages means that workers are initially hired with low wages but promised higher wages in the future, so the wage of workers increases over time. As a result, the strengths of the retention and insurance motives change over time. Around year 3, firms decide to backload wages slightly less in order to help workers accumulate precautionary savings. At this point, firms still get a positive value from the match and therefore want to retain workers but they are willing to let workers accumulate precautionary savings nevertheless. At year 5, the worker finds another job and receives a hiring bonus from the new firm, which leads to the jump in consumption and precautionary savings. This new firm also wants to retain its worker but the precautionary savings motive is now much stronger so new firms choose to attract the worker by offering insurance against unemployment risk in the form of frontloaded wages.

In conclusion, these results show that letting workers trade risk-free bonds leads to much richer life-cycle dynamics for wages, consumption and assets. We explore some of these implications with the quantitative model in section 4.

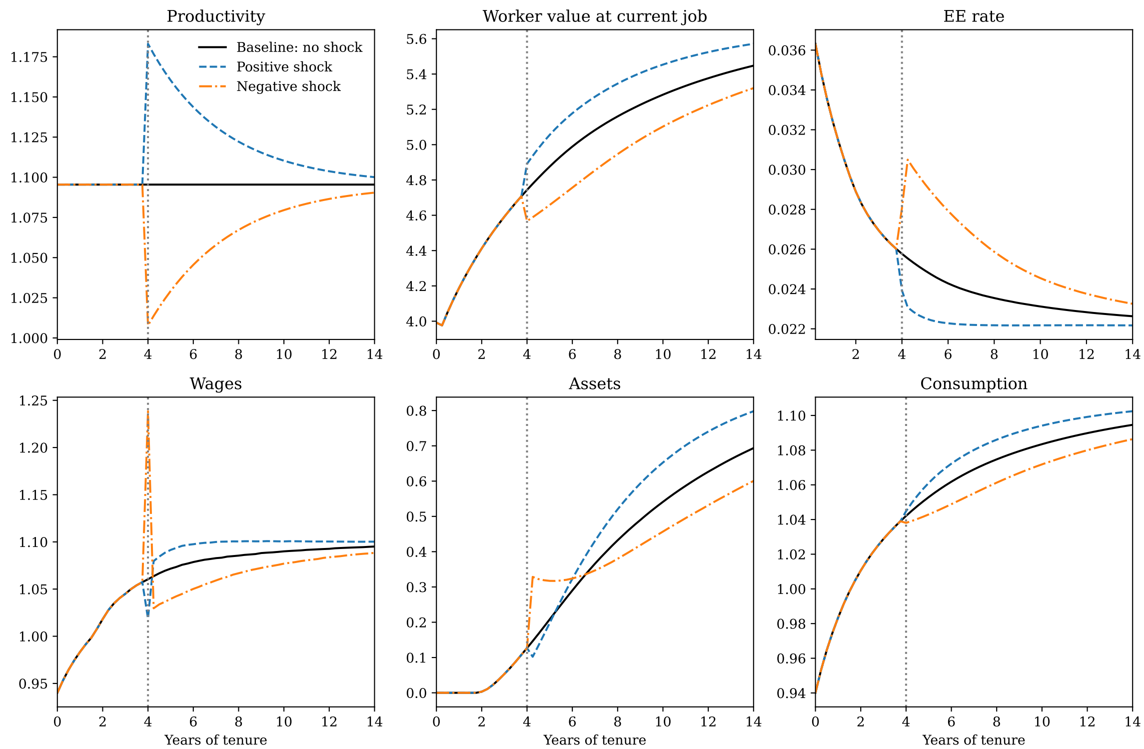
3.3 Implications for the pass-through of productivity shocks

This section describes how firms pass productivity shocks through to the wage, assets and consumption of workers. I focus here on describing how letting workers trade risk-free bonds influences the *dynamics* of the pass-through whereas section 4 will show that it also influences its *size*.

Figure 3 compares three scenarios for a worker who is hired from unemployment at $t = 0$ with no initial asset and happens to remain employed with the same firm throughout, in the quantitative model. In the first scenario described by the solid black line, productivity happens to remain constant throughout the match. In the second scenario described by the dashed blue line, productivity jumps up by one standard deviation after 4 years of tenure and then reverts back to its average level according to its persistence ρ estimated in the data. In the third scenario described by the dash-dotted orange line, productivity jumps down by one standard deviation after 4 years of tenure and then reverts back. The pass-through of a positive productivity shock is defined in this section as the difference between the dashed blue line from the second scenario and the solid black line from the first scenario. The pass-through of negative shocks is defined similarly.

Consider first the top panel of figure 3, which shows the paths of productivity x_t, of the worker continuation value at the current job W_t and of the worker EE expected separation rate p_t. After a positive shock to productivity, the firm chooses to increase the worker value at the current job in order to reduce the EE rate. Conversely, after a negative shock the firm chooses to reduce the worker value to increase the EE rate. The intuition behind this pass-through is similar to models with hand-to-mouth workers (e.g. see Souchier, 2023). Firms face a trade-off between insuring workers against risk and optimizing worker retention. On one hand, firms seek to insure workers against risk by keeping their consumption, and therefore their continuation value, independent of productivity shocks. On the other hand, firms seek to optimize workers retention by paying workers relatively more when productivity, and therefore profits, is high and relatively less when productivity is low. The optimal pass-through thus balances the worker's demand for insurance and the firm's desire to optimize worker retention.

Figure 3: The pass-through of productivity shocks to workers



Note: the top panel shows the paths for productivity x_t, worker value W_t and expected EE rate p_t for a worker hired at $t = 0$ from unemployment with zero assets. The bottom panel shows the paths of wages, assets and consumption. The paths are computed in the quantitative model where productivity is heterogeneous across firms and shocks are expected to occur. The dotted vertical line at 4 years shows when the productivity shock ν_t actually occurs. The solid black line represents the paths when the realization of productivity shocks ν_t happens to be null, the dashed orange line the paths when the realization of productivity is positive and the dash-dotted orange line when the realization is negative. Throughout, EE and EU shocks happen to be zero so the worker remains employed at the current firm.

The novel result here, and perhaps the most surprising, is how firms choose to implement this pass-through. This is shown in the bottom panel of figure 3, which shows the pass-through to wages w_t, assets a_t and consumption c_t. Consider the pass-through of *positive* shocks. The left panel shows that firms cut wages on impact and then increase wages substantially in the future. Meanwhile, workers deplete temporarily their assets to increase their consumption smoothly over time. Therefore, firms choose to increase the worker value after the shock with promises of future wage increases. Why would firm adopt such a strategy? Remember from section 3.2 that the path of wages over time depends on a trade-off between worker retention and insurance against unemployment risk. After a positive shock to productivity, the retention motive becomes stronger so firms choose to backload wages more, even if it means that workers are less insure against unemployment risk temporarily. In fact, firms backload wages so much that they fall on impact as in figure 3. Thus, after the shock firms seek to retain workers more not only by paying them more on average, but also by backloading their wages more. The persistence of shocks is critical for this result. If shocks were temporary, wages would instead increase only on impact. This is because backloading wages helps to retain workers in the future, which is only valuable if shocks are sufficiently persistent. In comparison, the model with hand-to-mouth workers does not feature these rich dynamics because firms do not change the degree of wage backloading in response to shocks.

The dynamics shown in figure 3 are quite stark, and not necessarily very realistic. Indeed, we do not usually see wages fall after positive productivity shocks in the data. These stark predictions however are due to the stylized nature of the model. For example, in a richer environment where firms have imperfect information about the persistence of shocks and respond to shocks as if they are temporary, wages smoothly rise over time in response to positive shocks[14]. Despite its stylized nature, the model from section 2 still sheds new light on existing empirical evidence. For instance, the model with assets is consistent with evidence that only persistent productivity shocks have persistent effects on wages whereas temporary shocks do not (Chan, Salgado and Xu, 2025). By contrast, even temporary shocks have persistent effects in models with hand-to-mouth workers because firms seek to smooth consumption over time. In section 4, I use the estimated model to compare the size of the pass-through for workers across the wealth distribution. Because of the complex dynamics shown in figure 3, I will focus in this exercise on the cumulative pass-through to wages and consumption because it is less sensitive to the exact timing of wage changes.

4 Quantitative implications

In this section, I estimate the model using matched employer-employee data from France. I then use the quantitative model to evaluate how the optimal degrees of wage backload-

[14] Other factors could make wages less likely to fall on impact after positive shocks. For example, in a model with a life-cycle firms might not choose to backload wage increases so much for older workers because they are about to retire, and for young workers because they have little assets. Similarly, in a model with financial frictions firms might not choose to backload wage increases so much because this is precisely when their financial constraint binds less.

ing and pass-through depend on the asset of workers, and show that in this respect the model is consistent with existing empirical evidence. Finally, I revisit the effects of a policy relaxing the worker's borrowing constraint and find that this policy improves not only the insurance that workers receive but also allocative efficiency.

4.1 Estimation

I estimate the model at quarterly frequency using matched employer-employee data from France between 2008 and 2019.

Data I combine annual data on firm balance sheet (FARE) with a panel of worker from social security data (DADS) containing 1/12th of the French labor force. I focus on private sector jobs in for-profit firms with at least 3 employees. I only keep in the sample workers with full-time jobs and permanent contracts, and prime age workers (25-55 years old). The final sample contains approximately 530,000 workers and 130,000 firms per year.

Estimation I estimate the model in two steps: first, I set some parameters externally; second, I infer the remaining model parameters by moment matching.

The model parameters set externally are the utility function, the foreign interest rate r and the matching function. The utility function is CRRA with coefficient $\gamma = 2$, following standard estimates from the macroeconomic literature. I set the interest rate to 1% quarterly. The matching function is Cobb-Douglas

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu} \phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). $B = 0.26$ is calibrated to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the job finding rate in my model. The cost of posting vacancy is modeled as

$$k(x_0) = k \exp\left(\phi(x_0 - 1)\right)$$

with $\phi = 10$.

The other model parameters are inferred by matching moments in the French data and in model-simulated data. Specifically, I simulate a panel of workers in the model and estimate the exact same set of moments in the model and in the data. The estimated parameters are the discount rate β, the vacancy posting cost k, the search efficiency on the job

Moments	**Data**	**Model**	**Parameters**	
Quarterly UE rate	21%	21.7%	Cost of posting vacancy k	0.2
Annual EE rate	6.3%	6.3%	On-the-job search efficiency κ	0.45
Annual EU rate	7%	7.0%	Separation rate δ	0.0195
Replacement ratio for unemp. benefits	62%	60%	Flow unemployment value b	0.7
Liquid assets/annual labor earnings	25%	25%	Relative discount factor $\beta(1+r)$	0.994
Variance of productivity growth	0.04	0.047	Volatility of productivity σ	0.08
Autocorrelation of order 1	- 0.22	- 0.24	Persistence of productivity ρ	0.93
Autocorrelation of order 2	- 0.06	- 0.068	Volatility of meas. error σ^{meas}	0.22

Note: the left panel shows the moments used in the estimation in the data and in the model. The right panel shows the parameters estimated internally. The UE, EE and EU rates and the moments on productivity are calculated using the French matched employer-employee data. The replacement ratio is calculated using data from the OECD on unemployment benefits. The liquid assets/annual labor earnings ratio is calculated using data from the Household Finance and Consumption Survey.

Table 1: Targeted moments in data vs. model and parameters

κ, the value of home production b, the exogenous separation rate into unemployment δ, the persistence of productivity ρ_x, its volatility σ_x and the volatility of i.i.d. measurement errors for annual productivity σ_x^{meas}. These 8 parameters are estimated using 8 moments in the data.

Table 1 shows the moments used in the estimation and the parameters. I use estimates of labor market flows (UE, EE and EU rates), of the replacement ratio for unemployment benefits and of liquid assets relative to labor income to discipline the mobility of workers across jobs, the risk that workers face and their access to financial markets. I use moments on firm productivity, measured as value added per worker, to discipline the risk that firms face and might transmit to workers. The UE, EE and EU rates are small relative to existing estimates in the literature, but this is not surprising given that they are measured for France and for workers with strong ties to the labor market. The replacement ratio for unemployment benefits is measured using OECD data, and is higher than existing estimates for the United States (e.g. Chodorow-Reich and Karabarbounis, 2016). The share of liquid assets relative to labor earnings is measured using estimates from the Household Finance and Consumption Survey.

The model is jointly estimated so all moments influence all parameters but the mapping between moments and parameters displayed in table 1 nevertheless gives some intuition about the estimation. In particular the amount of liquid asset is critical in pinning down the discount factor of workers relative to the interest rate $\beta(1+r)$. This new moment is standard in the precautionary savings literature (see Auclert, Rognlie and Straub, 2024), but novel for dynamic contract. If $\beta(1+r) = 1$ workers would accumulate an infinite amount of savings (as in Sotomayor, 1984, Chamberlain and Wilson, 2000) whereas when $\beta(1+r)$ is small, workers are relatively impatient and do not accumulate too much assets. A novel implication of this assumption in the context of optimal contracts is that it makes the promised value of workers drift over time while employed. This can be seen from equation (5) when workers cannot switch jobs ($\kappa = 0$), as the consumption of workers falls over time when $\beta(1+r) < 1$ even in the absence of EE separations.

Implications for contracts Table 2 shows key moment on wage contracts from the estimated model with assets and from a model with HtM workers. In the model with HtM workers, all parameters are kept constant except that $a_{t+1} = 0$ for employed and unemployed workers. The moments from table 2 are calculated from a panel of workers in model-simulated data. The wage and consumption growth are measured as the annual growth rate of wage and consumption for continuously employed workers. The pass-through of productivity shocks to wages and consumption are measured as the OLS coefficient of annual productivity growth on cumulative earnings growth over 2 years[15].

The first column of table 2 shows that wages grow on average by 1.9% per year in the model with assets and only by 0.6% per year in the model with HtM workers. This difference arises because firms take advantage of worker's existing assets to backload wages more in order to enhance worker retention. There are instances where firms choose to backload wages less in the model with assets to help workers self-insure (for example after EE separations), as explained in section 3.2, but on average wages are more backloaded when workers can trade risk free bonds. Workers use their assets to smooth their consumption over time so their consumption is more stable in the model with assets than with HtM workers (0.5% vs 0.6%), as shown in the second column.

The third column from table 2 shows that after a 10% increase in annual productivity, wages increase by an average of 5.4% in the next 2 years in the model with assets. By contrast, in the model with HtM workers, wages increase by only 1.29% so about 4 times less. This means that workers receive much less insurance from firms when they can trade risk-free bonds than when they are HtM. However, despite receive less insurance from firms, workers receive more insurance overall when they can trade risk-free bonds. This can be seen by looking at column 4, which shows that consumption responds by 0.88% after a 10% increase in productivity in the model with assets, and by 1.29% in the

[15] Specifically, I first compute the annual productivity and wage as the average within a firm-worker match across quarters, denoted $\overline{x}_y$ and $\overline{w}_y$ for year y. I then estimate the pass-through as

$$\theta^{w,x} \equiv \frac{\text{Cov}(\Delta \log \overline{x}_y, \Delta \log \overline{w}_y + \Delta \log \overline{w}_{y+1})}{\text{Var}(\Delta \log \overline{x}_y)} \tag{8}$$

The consumption pass-through $\theta^{c,x}$ is defined similarly.

	Wage growth	Consumption growth	Pass-through to w	Pass-through to c
Model with assets	1.9%	0.5%	54%	8.8%
Model with HtM workers	0.6%	0.6%	11.9%	11.9%

Note: the top row reports moments computed in the model with assets and the bottom row reports the same moments in a model with hand-to-mouth workers. The model with HtM workers is calibrated using exactly the same parameters as the baseline model, except that the constraint $a_{t+1} = 0$ is imposed for unemployed and employed workers. The wage and consumption growth are calculated annually for continuously employed workers. The pass-through to wages and consumption are computed as the regression coefficients of 2-year cumulative wage and consumption growth on annual productivity growth.

Table 2: Implications for wage contracts

model with HtM workers. The reason for this difference is that firms do not need to smooth wages across states when workers can smooth consumption themselves using their existing assets so they pass productivity shocks through relatively more to optimize worker retention. The marginal propensity to consume implied by the model can be computed as the ratio of the consumption pass-through to the wage pass-through. It is equal to $8.8/54 = 16\%$ in the model with assets and $11.9/11.9 = 100\%$ in the model with HtM workers, which highlights that workers receive significant insurance outside firms against productivity shocks.

Taken together, these results show that workers use financial markets to smooth considerably their consumption over time and across states. In turns, the insurance that workers receive outside firms significantly crowds out the insurance that workers receive inside firms in the sense that firms smooth wages much less over time and across states when workers have access to financial markets[16]. The fact that this crowding out is large is one of the main lessons from the quantitative exercise, and it will also be at work when we compare wage contracts across the wealth distribution in sections 4.2 and 4.3 and when we relax borrowing constraints in section 4.4.

Validation The model is broadly consistent with empirical evidence on wage growth and pass-through, which were not targeted in the estimation. In particular, the tenure profile for wages, which measures the degree of wage backloading, is almost identical in the model (18%) and in the data (14%)[17]. My estimate for the pass-through of productivity shocks to wages is between those typically measured using administrative data and

[16]To illustrate the size of the crowding out, consider the following back-of-envelope calculation: if letting workers trade risk-free bonds did not influence wage contracts, the pass-through would have remained equal to 11.9% in the model with assets; the pass-through to consumption would therefore have been approximately equal to $11.9\% \times \text{MPC} = 11.9\% \times 16\% = 1.9\%$, which is less than a quarter the pass-through in the model with assets (8.8%). The same back-of-envelope calculation can be done for wage growth.

[17]The tenure profile is measured as the cumulative wage growth after 25 years of tenure at the same firm. In the data, the tenure profile is measured after controlling for overall experience in the labor market using a polynomial in experience.

statistical models of earnings (e.g. Guiso, Pistaferri and Schivardi, 2005) and those measured using exogenous shocks to firm productivity (e.g. Kline, Petkova, Williams and Zidar, 2019)[18]. In sections 4.2 and 4.3, I show that the model is also consistent with existing empirical evidence that wage growth and wage pass-through vary across the wealth distribution. Finally, the model-implied marginal propensity to consume ($8.8/54 = 16\%$) is much smaller than 1 but larger than 0, consistent with existing empirical evidence.

4.2 Wealth at birth and life-cycle earnings inequality

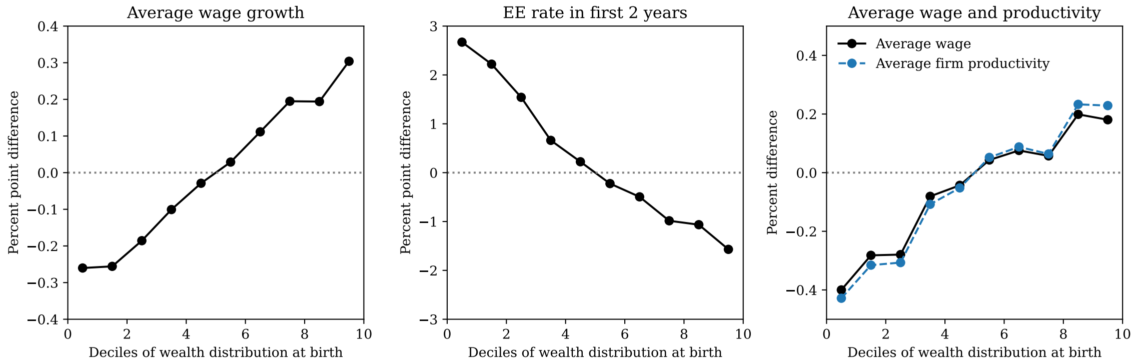
I use the model to evaluate how the optimal degree of *wage backloading* depends on the worker's assets, and show that workers born rich select jobs where wages are more backloaded but also pay more on average.

Figure 4 shows how the average wage growth, the EE rate and the average wage and productivity depend on the worker's wealth "at birth". To compute this figure, I simulate the career of a panel of workers starting unemployed with levels of assets corresponding to different deciles of the wealth distribution[19]. The asset workers hold at the start of the simulation is their wealth "at birth". This exercise captures, in reduced form, the different trajectory of workers who start their career with different initial assets and therefore experience different life-cycle income profiles. The figures are normalized so the median equals 0.

[18] Estimates using statistics model of earnings typically range between 3% and 12% across countries but these are subject to downward bias due to measurement errors. Estimates using identified shocks are typically much larger but these estimates are measured in specific markets not representative of my sample. For example, Kline et al. (2019) study the market of patent inventors in the United States.

[19] I assume that workers find a job in the first period to better emphasize the impact of initial asset holdings on wage contracts.

Figure 4: Wealth at birth and life-cycle earnings inequality



Note: the statistics are computed by simulating a panel of workers starting from unemployment at $t = 0$ with wealth drawn from the stationary distribution. The wealth that workers have at the start of the distribution is their "wealth at birth". The left panel shows the average annual growth rate of wages, the middle panel shows the average EE rate of workers during the first 2 years after they leave unemployment and the right panel shows the average wage of workers and average productivity of matches. The figures are normalized so that the median equals 0.

The left panel shows that relatively wealthy workers, who start their career with more assets, experience higher wage growth than relatively poor workers, who start their career with less assets (e.g. the top decile experiences an average annual wage growth of 1.4% compared to 0.9% for the bottom decile, a difference of 0.5 percent points). The reason is that wealthy workers receive wages that are more backloaded so their wages growth faster on the job. The consumption of wealthy workers, however, is more stable over time than the consumption of poor workers (not shown in figure 4). The middle panel shows the cumulative EE rate within the first 2 years since workers first match with their employers. The rate is 10.7% for workers at the bottom decile as opposed to 7.1% for workers at the top decile, a difference of 3.6 percent points. The reason why relatively rich workers are less likely to switch jobs is precisely because their wages are more backloaded. Finally, the right panel shows the average wage in solid black and average productivity in dotted blue for workers across the wealth distribution. Relatively rich workers receive average wages 0.55% higher than poor workers, and they are matched with firms that are 0.65% more productive on average. The reason is that firms' expected profits increase when they are more likely to retain workers. Because of the free entry condition, these firms offer workers higher average wages. Besides, firms are more likely to invest in better technology x_0 because they are less concerned about losing their workers. Another way to say this is that the hold-up problem that prevents firms from investing optimally in productivity x_0 is lessened for wealthy workers because these workers effectively pay the upfront investment in technology x_0 themselves with backloaded wages. Thus, income and productivity inequality here arise because poor workers are further away from the first-best level of investment than wealthy workers are. Taken together, figure 4 shows that wealthy workers select jobs where wages are more backloaded but that pay more on average in equilibrium. As a result, firms know that they can retain these workers better and choose to invest more in productivity. By contrast, poor workers select jobs with more stable earnings because they cannot smooth consumption themselves but these jobs also pay less and have lower productivity[20].

This mechanism is consistent with evidence on earnings growth across workers documented in the literature. In particular, Guiso et al. (2012) find using Italian data that firms that become financial constrained are more likely to backload the wages of workers who are more likely to be wealthy, namely managers and white-collar workers relative to blue-collar workers. Halvorsen et al. (2022) use administrative data from Norway and find that the children of parents with high net wealth experience higher wage growth than children of parents with low net wealth. The model also provides a new explanation for the widely documented persistence of income inequality across generations (e.g. Solon, 1992) in that parents with high earnings are more likely to be wealthy, thus allowing their kids to select jobs with wages that are more backloaded but also higher on average.

4.3 Insurance against productivity shocks over the wealth distribution

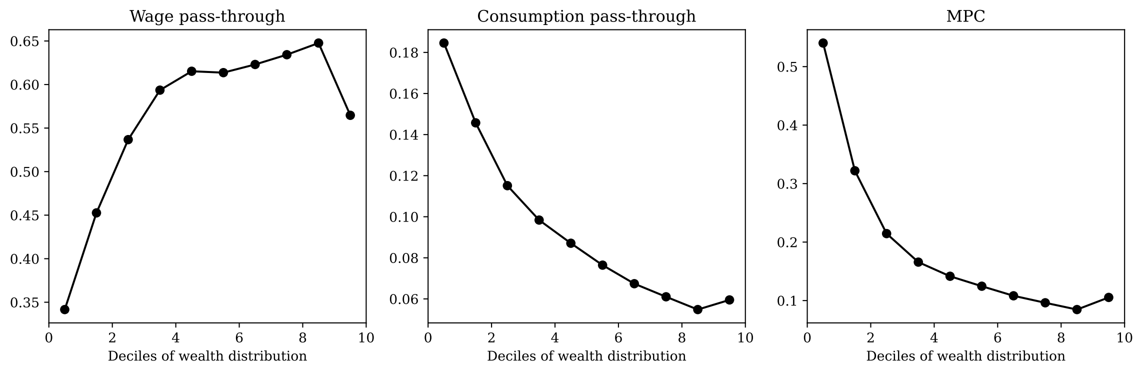
I use the model to evaluate how the optimal degree of *pass-through* depends on the worker's assets, and show that wealthy workers receive less insurance from firms but more insurance overall against these shocks.

Figure 5 shows the pass-through, defined in equation (8), for different quantiles of assets in the previous year. To compute this figure, I simulate a panel of workers and compute the pass-through in the stationary distribution. The left panel shows that the pass-through to wages increases over the wealth distribution, except for the top decile. The middle panel shows that the pass-through to consumption has the opposite pattern and is larger for relatively poor workers. Finally, the right panel shows that the implied marginal propensity to consume, defined as the ratio of the pass-through to consumption over the pass-through to wages, falls with assets. The results from figure 5 thus show

[20]This mechanism complements the one described in Eeckhout and Sepahsalari (2023) and Chaumont and Shi (2022) with directed search and fixed-wage contracts. In these papers, wealthy workers receive higher wages on average because they choose to search in markets with a lower job finding rate but higher average wages and productivity. This mechanism is also at play in my model but it is reinforced by the ability of firms to backload wages. A key difference is that models with fixed-wage contracts imply that poor workers experience relatively high wage growth relative to wealthy workers because they start from the bottom of the job ladder. By contrast, with optimal contracts wealthy workers are the ones who experience higher wage growth as suggested in the data.

that workers along the wealth distribution receive different mix of insurance inside and outside the firm. Relatively poor workers receive more insurance inside the firm (their wage pass-through is low and their MPC is high) whereas relatively rich workers receive more insurance outside the firm (their wage pass-through is high but their MPC is low).

Figure 5: Heterogeneous pass-through over the wealth distribution



Note: the statistics are computed for each bin of the wealth distribution from the previous year. The pass-through to wages and consumption are computed as in table 2. The MPC is calculated as the ratio of the pass-through to wages to the pass-through to consumption.

Why won't firms insure relatively poor workers even more? The reason is that relatively poor workers tend to be at the bottom of the job ladder. As a result, they are more likely to switch jobs and receive lower wages from firms. Firms thus find it optimal to increase their consumption relatively more in response to positive productivity shocks to induce them to stay, and reduce their consumption relatively more in response to negative productivity shocks to let them go. By contrast, firms are willing to provide more insurance overall to rich workers because they are less likely to switch jobs and they generate less profit to firms.

The results from figure 5 are consistent with existing evidence from the literature on pass-through and marginal propensity to consume. Specifically, Fagereng, Guiso and Pistaferri (2017) show that the pass-through of firm-level productivity shocks to wages is increasing in assets using data from Norway. Remarkably, in their sample the pass-through is about twice larger for the top decile than it is for the bottom decile, which is consistent with figure 5. Finally, the fact that the marginal propensity to consume is decreasing with liquid assets is also consistent with a broad set of evidence.

Experience < 10 years	**Growth w**	**Growth c**	**Pass-through w**	**Pass-through c**	**Avg. w**	**Avg. x**
Baseline ($\underline{a} = 0$)	1.2%	1.4%	48%	14%	1.089	1.18
Counterfactual ($\underline{a} = -2$)	4.9%	0.4%	68%	10%	1.096	1.20

Note: the top row considers the baseline model without borrowing whereas the bottom row considers a model calibrated with the same parameters except that workers can borrow up to $\underline{a} = -2$, which corresponds to about 2 quarters of average labor income. The statistics are computed for workers with less than 10 years of experience, which means that they were unemployed with no asset less than 10 years ago. The first two columns report the average growth rate of wages and consumption. The next two columns report the pass-through of wages and consumption, computed as in table 2. The last two columns report the average wage of workers and the average productivity of matches.

Table 3: Implications of relaxing borrowing constraints

4.4 Public insurance policies: relaxing the borrowing constraint

The previous sections showed that workers across the wealth distribution receive different amounts of insurance inside and outside firms. In this section, we show that policies that relax borrowing constraints have a similar effect in that it improves the insurance that workers receive outside firms. As a result, workers who can now borrow receive less insurance from firms, but more insurance overall, and they end up being better matched with firms and receive higher average wages. In this sense, letting poor workers borrow makes them similar to wealthy workers.

Table 3 presents the result of this counterfactual exercise. The first line shows the baseline model where workers cannot borrow whereas the second line shows the counterfactual economy where workers can can borrow at rate r up to -2, which corresponds to approximately 2 quarters of labor earnings. I interpret this counterfactual as arising from a policy improving worker's access to borrowing, such as subsidizing loans to workers. In both the baseline and the counterfactual, I focus on workers with less than 10 years of labor market experience, meaning workers who were unemployed with 0 assets less than 10 years ago[21]. I focus on this group because workers with more labor market experience have the time to accumulate precautionary savings and are thus less affected by this policy. In the counterfactual economy, all the model parameters are kept constant except the borrowing constraint.

Table 3 shows that workers receive less insurance from firms but more insurance overall when the borrowing constraint is relaxed. This can be seen along 2 dimensions. First, the growth of wages is much higher (4.9% compared to 1.2%) but the growth rate of consumption is much lower (0.4% compared to 1.4%). The reason is that workers select jobs

[21]Focusing on workers who start from unemployment with 0 assets is a parsimonious way of capturing the life-cycle of workers without having to model it explicitly.

with backloaded wages when they can borrow, and then use their ability to borrow to smooth their consumption. Second, the pass-through of productivity shocks to wages is much higher (68% compared to 48%) but the pass-through to consumption is much lower (10% compared to 14%). The reason is that workers select jobs with more volatile earnings when they can borrow because they can smooth their consumption by borrowing. In fact, the implied marginal propensity to consume falls from 28% to 14% when workers can borrow. Overall, the crowding out of the insurance inside the firm by the insurance outside the firm is large, as in table 2. The most surprising result from table 3 is that workers are matched with more productive firms (1.2 compared to 1.18) and receive higher average wages (1.096 compared to 1.089) when the borrowing constraint is relaxed. The reason is that allowing workers to borrow enables them to search for jobs that have more backloaded wages, as in section 4.2. Firms invest in better technology in these jobs because they are less worried about loosing their workers to competing firms. As a result, workers receive higher wages on average.

In conclusion, relaxing borrowing constraints improves the insurance that workers receive overall and improve matching efficiency. Workers however should expect to receive wages that are more backloaded and more volatile after this policy is implemented. Conducting this counterfactual exercise in a precautionary savings model without optimal contract would lead to overstate the benefits in terms of insurance, but understate the benefits in terms of allocative efficiency.

5 Hidden assets

Until now, I have assumed that the asset of workers is public information to firms. This assumption implies that firms know the assets that workers have when they match, and that firms and workers can contract on the saving decision of workers when they are employed. These implications are not very realistic, so it is natural to wonder whether the allocation would be very different if assets were private information to workers.

Private information about assets raises two issues, which have been partially studied in the literature. First, workers might choose to direct their search in markets (v, a) even though their current asset is not a. This is an issue because firms expect workers to have assets a when they design wage contracts. This has been studied with fixed-wage contracts in Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023), but not with optimal contracts. Second, the saving decision of employed workers might differ from the recommended policy in the contract. This issue has been extensively studied in the unemployment insurance literature (e.g. Werning, 2002, Abraham and Pavoni, 2008).

In this section, I characterize the deviations of workers when assets are private information. Specifically, I ask what deviation would a worker follow if she is offered the contracts designed under the assumption that assets are public information, while in fact the worker can lie about her existing assets when she matches with a firm and about her saving decision while employed. I first show that with CARA utility, there is no profitable deviation for workers so that the allocation is identical with hidden assets and publicly observable assets. I then show that workers benefit from deviation when utility if CRRA because of wealth effects on search. Throughout, I will emphasize results that differ from those already derived in the literature.

5.1 Equivalence with CARA utility: no wealth effect on search

I first show that with CARA utility $u(c) = -\gamma \exp(-\gamma c)$, the optimal contract when assets are public is also optimal when assets are private information to workers. This result is important because it shows that the assumption that assets are public information is not that critical for the analysis. In particular, we can interpret the model as one where firms propose a set of contracts (wages conditional on history of shocks and tenure) to workers, and where workers select the contracts they prefer depending on their assets and choose to save and consume independently of firms.

Proposition 3. *Assume that utility is CARA and that workers face no borrowing constraint. Then, the equilibrium allocation with hidden assets is exactly identical than the allocation with publicly observable assets.*

Proof. See appendix A.5. □

This equivalence between private and public information arises because of the absence of wealth effect on search with CARA utility and it relies on two results. First, in the pseudo Euler equation 7 the terms capturing wealth effects is null, that is $\mathcal{W}_t = 0$. This result, extensively studied in the literature on optimal unemployment insurance, arises because two workers with the same wage contract but different levels of wealth are equally likely to switch job. Thus, the optimal contract with observable savings decision satisfies the Euler equation, and therefore it solves the optimal contract with hidden savings conditional on the initial asset of workers being known to firms.

Second, the absence of wealth effects means that workers have no incentive to report a different level of asset than their actual assets when they first match with firms. In particular, a worker with asset a never wants to under-report or over-report her assets by searching for a job in a market $(v, \tilde{a})$ where $\tilde{a} \neq a$. To see why, it is useful to assume that workers do not face any borrowing constraint first. In this case, it turns out that in any market (v_1, a_1) and (v_2, a_2) such that $v_1 \exp(\gamma r a_1) = v_2 \exp(\gamma r a_2)$, the job finding rate will be the same and workers will be offered the exact same contract. Thus, any deviation to a market $(\tilde{v}, \tilde{a})$ from (v, a) will yield the same value to workers than a deviation to a market $(\tilde{v} \exp(\gamma r(\tilde{a} - a)), a)$, i.e. a market with a different promised value but the true asset. But then, searching in such a market was already feasible for workers and was not optimal since (v) solves the search decision of workers in (2) conditional on their asset a. Therefore, workers would not benefit from deviating to a market where firms expect to meet workers with $\tilde{a}$ when their actual asset is a.

When workers face borrowing constraints, they too cannot benefit from reporting a level of asset $\tilde{a}$ different than their actual asset a. To see why, remember that firms take into account the worker's borrowing constraint when they design wage contract. When they expect it to bind, they do not backload wages as much because they know that workers cannot smooth consumption as much by consuming their existing assets. Thus, a worker with a lot of assets would receive a path of wages that is less backloaded by pretending to have very little assets. However, this worker would also receive lower average wages because firms generate lower expected profits in these markets. Overall, the worker is worse off. Conversely, a worker would not benefit by pretending to have more assets than she actually have since in this case she would enjoy higher average wages but her consumption would be more backloaded. One way to interpret this result is by remembering that assets are used in the optimal contract as a substitute for the commitment power of workers. Ex-ante, workers would be better off if they could commit to transfers after EE separations. Thus, they have no incentive to under-report their assets.

5.2 Optimal deviations with CRRA

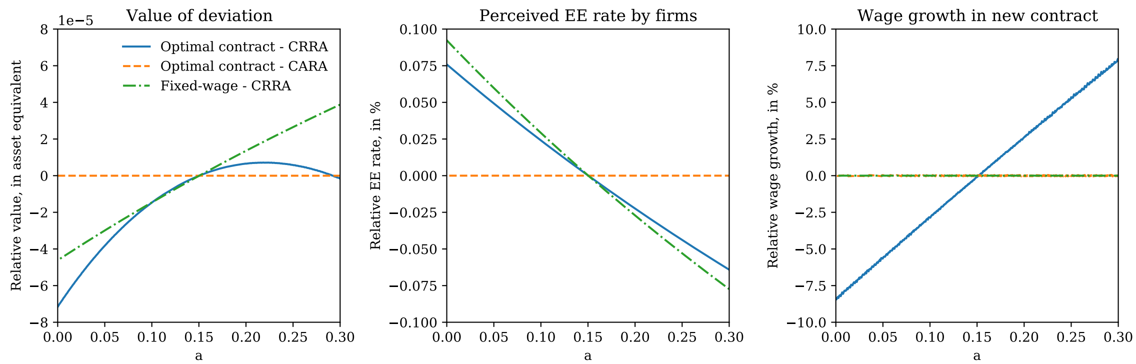
I now show that the optimal contract when assets are public information is no longer incentive compatible when assets are private information and utility is CRRA. Specifically, I first solve for the optimal contract assuming that firms observe the worker's assets perfectly. I then ask how a worker would choose to deviate if she is offered this set of wage contracts. This exercise does not characterize the equilibrium with hidden assets but it helps assess whether the assumption that assets are public information is critical.

I first only briefly describe deviations in terms of savings that workers make while they are employed, since these have been extensively studied in the literature on optimal unemployment insurance. Equation (7) shows that when assets are public information, firms choose them to smooth the worker's consumption and to influence search through wealth effects $\mathcal{W}_t$. In particular, as explained in section 3.2, firms will make workers save more than the Euler equation would recommend when they want to retain them because this reduces the EE rate of workers. When the saving decision of workers is private information, the optimal deviation of workers is to follow their Euler equation, thus consume more today and save less. This deviation will make them more likely to switch jobs in the next period relative to what the firm would prefer.

I now describe the novel issue that arises in the context of wage contracts. Specifically, workers might deviate by searching in markets where the asset they are supposed to have $\tilde{a}$ is different than the asset they actually have a. The left panel of figure 6 illustrates this deviation by showing how much a worker with asset $a = 0.15$ would gain by searching in a market with $\tilde{a} \neq a$. For simplicity, the deviation is computed in a version of the model where workers only live for 2 periods, where they face no borrowing constraint and where productivity is constant across and within matches. I also compute contracts assuming that the saving decision is private information to stress the role that the initial asset of workers play[22]. The utility gains are measured in asset equivalent, that is how much additional assets would be needed to achieve the same gain than the deviation. The blue line shows that workers benefit by searching in markets with higher assets, thus pretending to be wealthier. In this example, the optimal deviation for a worker with asset $a = 0.15$ is to search in markets indexed by $\tilde{a} = 0.22$. By contrast, the orange line confirms that workers do not benefit by deviating when utility is CARA. Finally, the green line shows the value of a deviation with CRRA utility but when firms are restricted to offer fixed-wage contracts, as in Chaumont and Shi (2022) and Eeckhout and Sepahsalari (2023). In this case, workers also benefit from searching in markets with higher assets $\tilde{a}$ but the value from deviations is monotonically increasing in $\tilde{a}$.

[22] I solve the optimal contract with hidden savings using the first-order approach, that is adding the Euler equation as a constraint on the optimal contract.

Figure 6: Optimal deviation with hidden assets



Note: the figure considers how workers would deviate from the equilibrium computed under the assumption that their initial asset is observable when their asset is actually private information. The x-axis shows the asset reported by workers. This exercise is done in a 2-period model where workers are employed in period 1 and can switch jobs in period 2. The left panel reports the value from a deviation for workers relative to reporting their true level of asset, $a = 0.15$. This panel shows that workers benefit by overstating their assets whenever their preference is CRRA. The middle panel shows how firms perceive the EE rate of workers as a function of their initial asset. This panel shows that firms perceive workers to be less likely to leave when they are wealthy, which is why they are willing to pay them higher average wages. The right panel shows the average wage growth of workers. This panel shows that workers receive more backloaded wages when they report higher assets, but only under optimal wage contracts. Taken together, this figure shows that workers benefit by overstating their asset because firms perceive them as less likely to leave and therefore are willing to pay them higher wages. However, the gains from deviating upwards are not monotonic with optimal contracts because the wage of workers becomes too backloaded when they pretend to be more wealthy than they actually are.

What accounts for these differences? The middle panel of figure 6 shows why workers benefits by deviating upward when utility is CRRA. It shows the EE rate of workers with different assets *a keeping the wage contract constant*, relative to the EE rate of workers with $a = 0.15$. By keeping the wage contract constant, we can evaluate how firms perceive the likelihood that they can retain workers as a function of the assets of workers. The figure confirms that workers are less likely to leave when their initial assets are high because of wealth effects on search: relatively rich workers search in markets with a relatively high value and low job finding rate. Because workers are perceived by firms as less likely to leave, the expected profit from matches rises. Because of the free entry condition, workers end up receiving higher average wages in these markets. This is the reason why workers benefit by searching in markets designed for workers with higher assets.

The right panel of figure 6 shows why the gains from deviating are not monotonic with optimal contract, whereas they are with fixed-wage contracts. Specifically, the right panel shows the growth rate of wages that workers receive in markets with different assets a. Because workers are perceived to be less risk-averse when they report higher initial assets a, firms optimally choose to backload wages more. This makes these contracts relatively unattractive to workers with lower assets because they would prefer consumption to be smoother over time. As a result, workers choose to deviate upward but not too much. By contrast, when workers are offered fixed-wage contracts, they do not face this cost of deviating, and therefore the benefit from reporting higher assets is always increasing.

Thus, these results show that the optimal contract described in section 3 and 4 is not incentive compatible when assets are private information. However, deviations by workers are bounded when contracts are optimal and the gains from deviating appear to be very small in this 2-period example (10^{-5}).

6 Conclusion

This paper builds a new model with optimal wage contracts, assets and search frictions. The insurance that workers receive outside the firm, through financial markets, significantly crowds out the insurance that they receive inside the firm, through optimal wage contracts. As a result, wealthy workers receive wages that are more backloaded are more volatile relative to poor workers, but they are also matched with more productive firms and receive higher average wages. This model has novel implications for policies that relax borrowing constraints because these policies enable poor workers to receive wage contracts that are similar to those received by wealthy workers. The model built in this paper is the first to combine optimal wage contracts with realistic financial markets. As such, it can be used as a foundation for future work studying how business cycles influence the earnings and consumption of different workers.

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Appendix

Contents

A Model appendix

A.1 Proof of proposition 1

A.1.1 Part 1

Consider any allocation that solves the optimal contract with a path for assets $a_{t+1}(x^t)$, where x^t denote the history of productivity since the beginning of the match. We can construct a new path wages $\tilde{w}_t(x^t)$ and transfers $\tilde{\tau}_t^{ee}(x^t)$, $\tilde{\tau}_t^{eu}(x^t)$ such that the allocations $c_t(x^t)$, $p_t(x^t)$ remain the same and solve the optimal contract while the path for assets satisfies $\tilde{a}_{t+1}(x^t) = 0$.

Let the new paths of transfers be $\tilde{\tau}_t^{ee}(x^t) = \tau_t^{ee}(x^t) - a_t(x^{t-1})$ and $\tilde{\tau}_t^{eu}(x^t) = \tau_t^{eu}(x^t) + a_t(x^{t-1})$. The new path of wages is $\tilde{w}_t(x^t) = (1+r)a_t(x^{t-1}) + w_t(x^t) - a_{t+1}(x^t)$ for all periods except the first, and $\tilde{w}_t = w_t - a_{t+1}$ for the first period. This wage ensures that the path of consumption $c_t(s^t)$ can remain the same under the new contract.

First, we show that the worker value remains constant after any history with this new contract. The worker value satisfies

$$\begin{aligned}\tilde{V}_t(x^t) = &\ \delta U(\tilde{\tau}_t^{eu}(x^t)) + (1-\delta)\left[\tilde{W}_t(x^t) + S(\tilde{W}_t(x^t), -\tilde{\tau}_t^{ee}(x^t))\right] \\ = &\ \delta U(a_t(x^{t-1}) + \tau_t^{eu}(x^t)) + (1-\delta)\left[\tilde{W}_t(x^t) + S(\tilde{W}_t(x^t), a_t(x^{t-1}) - \tau_t^{ee}(x^t))\right]\end{aligned}$$

with $\tilde{W}_t(x^t) = u(c_t(x^t)) + \beta \mathbb{E}_{x_{t+1}}\left[\tilde{V}_{t+1}(x^{t+1})|x^t\right]$. This shows that $\tilde{V}_t(x^t) = V_t(x^t)$ and $\tilde{W}_t(x^t) = W_t(x^t)$ after any x^t.

Then, we show that this new contract achieves the same *initial* value to firms (note that the firm value will differ after some histories). The firm value after history x^t under the original contract satisfies

$$
\begin{aligned}
\Pi_t(x^t) = &\quad (1-\delta)\left(1-p(W_t(x^t), a_t(x^{t-1})-\tau_t^{ee}(x^t))\right)\left(x_t - w_t(x^t) + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right)\\
&\qquad -\delta(1+r)\tau_t^{eu}(x^t) + (1-\delta)p(W_t(x^t), a_t(x^{t-1})-\tau_t^{ee}(x^t))(1+r)\tau_t^{ee}(x^t)\\
= &\quad (1-\delta)\left(1-p(W_t(x^t), -\tilde{\tau}_t^{ee}(x^t))\right)\left(x_t - \tilde{w}_t(x^t) + (1+r)a_t(x^{t-1}) - a_{t+1}(x^t) + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right)\\
&\qquad -\delta(1+r)\left(\tilde{\tau}_t^{eu}(x^t) - a_t(x^{t-1})\right) + (1-\delta)p(W_t(x^t), -\tilde{\tau}_t^{ee}(x^t))(1+r)\left(\tilde{\tau}_t^{ee}(x^t) + a_t(x^{t-1})\right)
\end{aligned}
$$

Now define $\tilde{\Pi}_t(x^t) = \Pi_t(x^t) - (1+r)a_t(x^{t-1})$ for all x^t. Use $\tilde{W}_t(x^t) = W_t(x^t)$ in the previous equation to get

$$
\begin{aligned}
\tilde{\Pi}_t(x^t) = &\quad \Pi_t(x^t) - (1+r)a_t(x^{t-1})\\
= &\quad (1-\delta)\left(1-p(\tilde{W}_t(x^t), -\tilde{\tau}_t^{ee}(x^t))\right)\left(x_t - \tilde{w}_t(x^t) + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x^t\right]}{1+r}\right)\\
&\qquad -\delta(1+r)\tilde{\tau}_t^{eu}(x^t) + (1-\delta)p(\tilde{W}_t(x^t), -\tilde{\tau}_t^{ee}(x^t))(1+r)\tilde{\tau}_t^{ee}(x^t)
\end{aligned}
$$

which shows that $\tilde{\Pi}_t(x^t)$ is precisely the firm value under the alternative contract.

Now plug this in the problem of new entrants to verify that the firm value is the same in the first period

$$
\begin{aligned}
\tilde{\Pi}_0(v, a_t, x_0) = &\quad x_0 - \tilde{w}_t + \frac{\mathbb{E}_{x_{t+1}}\left[\tilde{\Pi}_{t+1}(x^{t+1})|x_0\right]}{1+r}\\
= &\quad x_0 - w_t + a_{t+1} - a_{t+1} + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x_0\right]}{1+r}\\
= &\quad x_0 - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi_{t+1}(x^{t+1})|x_0\right]}{1+r}\\
= &\quad \Pi_0(v, a_t, x_0)
\end{aligned}
$$

Therefore, the new contract with $a_{t+1}(x^t) = 0$ after any x^t achieves the same initial firm value. The allocations satisfies the constraints on the optimal contract every period, including the borrowing constraint, so the new contract is also a solution to the optimal contract. Finally, the EE separation rate $p_t(x^t)$ remains the same since

$$
p(\tilde{W}_t(x^t), -\tilde{\tau}_t^{ee}(x^t)) = p(W_t(x^t), a_t(x^{t-1}) - \tau_t^{ee}(x^t))
$$

A.1.2 Part 2

I prove part 2 under the assumption that productivity is constant within matches so I will denote the wage as w_t, where t denotes time. Denote the allocation achieved with transfers as c_t^*, p_t^* and the corresponding transfers as $(\tau_t^{ee})^*, (\tau_t^{eu})^*$. We assume that $a_{t+1}^* = 0$, which is without loss of generality from part 1.

We now construct a new contract consisting of paths for wages, assets and transfers $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ such that

a) $\tau_t^{ee} = 0$ for all t,

b) the worker consumption c_t and values V_t, W_t are the same for all t,

c) the worker EE separation rate $p_t \equiv p(W_t, a_t - \tau_t^{ee})$ is the same for all t,

d) the initial firm value $\Pi_0(v, a, x_0)$ is the same.

This new contract delivers the same value to firms and satisfies all the constraints of the relaxed problem in which transfers τ_t^{ee} can be implemented (note that there is no borrowing constraint in part 2). Since this contract also satisfies the constraint $\tau_t^{ee} = 0$, it must solve the optimal contract where transfers τ_t^{ee} cannot be implemented. Thus, it would show that we can implement the same allocation with a contract that satisfies $\tau_t^{ee} = 0$ provided that workers can use assets.

Construct the new contract $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ using

$$w_t = w_t^* - (1+r)a_t + a_{t+1}$$

$$a_{t+1} = (\tau_{t+1}^{ee})^*$$
$$\tau_t^{ee} = 0$$
$$\tau_t^{eu} = (\tau_t^{eu})^* - a_t$$

except for the first period where $w_t = w_t^* + a_{t+1}$. The consumption of the worker implied by this path is unchanged since

$$c_t = (1+r)a_t + w_t - a_{t+1} = w_t^* = c_t^*$$

for all periods except the first, and

$$c_t = (1+r)a_0 + w_t - \tilde{a}_{t+1} = (1+r)a_0 + w_t^* = c_t^*$$

for the first period.

We now show that the worker value remains constant after any history with this new contract. The worker value satisfies

$$\begin{aligned} V_t = &\quad \delta U(a_t + \tau_t^{eu}) + (1-\delta)\left[W_t + S(W_t, a_t)\right] \\ = &\quad \delta U((\tau_t^{eu})^*) + (1-\delta)\left[W_t + S(W_t, (\tau_t^{ee})^*)\right] \end{aligned}$$

with $W_t = u(c_t^*) + \beta V_{t+1}$. This shows that $V_t = V_t^*$ and $W_t = W_t^*$ for all t. It also follows that the EE rate p_t remains the same since

$$p_t \equiv p(W_t, a_t) = p(W_t^*, \tau_t^{ee})$$

Finally, consider the firm value with these paths, denoted Π_t. It satisfies

$$\begin{aligned} \Pi_t = &\quad (1-\delta)\left(1 - p(W_t, a_t)\right)\left(x_0 - w_t + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)\tau_t^{eu} \\ = &\quad (1-\delta)\left(1 - p(W_t^*, (\tau_t^{ee})^*)\right)\left(x_0 - w_t^* + (1+r)a_t - a_{t+1} + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)(\tau_t^{eu})^* + \delta(1+r)a_t \\ = &\quad (1-\delta)\left(1 - p(W_t^*, (\tau_t^{ee})^*)\right)\left(x_0 - w_t^* - a_{t+1} + \tfrac{\Pi_{t+1}}{1+r}\right) - \delta(1+r)(\tau_t^{eu})^* \\ &\quad +(1+r)a_t + (1-\delta)p(W_t^*, (\tau_t^{ee})^*)(1+r)(\tau_t^{ee})^* \end{aligned}$$

Now guess and verify that $\Pi_t = \Pi_t^* + (1+r)a_t$. Going back to the first period, we get

$$\begin{aligned} \Pi_0(v, a_t, x_0) = &\quad x_0 - w_t + \tfrac{\Pi_{t+1}}{1+r} \\ = &\quad x_0 - w_t^* - a_{t+1} + a_{t+1} + \tfrac{\Pi_{t+1}^*}{1+r} \\ = &\quad x_0 - w_t^* + \tfrac{\Pi_{t+1}^*}{1+r} \\ = &\quad \Pi_0^*(v, a_t, x_0) \end{aligned}$$

and therefore the initial value of the firm is the same with this new contract.

A.1.3 Part 3

The proof is similar to part 2 except that the new contract satisfies $\tau_t^{eu} = 0$ for all t. The new contract $w_t, a_{t+1}, \tau_t^{ee}, \tau_t^{eu}$ satisfies

$$w_t = w_t^* - (1+r)a_t + a_{t+1}$$

$$a_{t+1} = (\tau_{t+1}^{eu})^*$$
$$\tau_t^{ee} = (\tau_t^{ee})^* + a_t$$
$$\tau_t^{eu} = 0$$

except for the first period where $w_t = w_t^* + a_{t+1}$. From there, the proof is identical to part 2.

A.2 Consumption growth condition (5)

Consider the optimal contract with $\tau_t^{ee} = \tau_t^{eu} = 0$ and denote the Lagrange multipliers on the constraints as $\eta_t, \lambda_t, \mu_t, \zeta_t$. The optimality conditions are

$$\begin{aligned}
w_t : \quad & (1-\delta)(1-p(W_t,a_t)) = \mu_t \\
c_t : \quad & \lambda_t u'(c_t) = \mu_t \\
V(x_{t+1}) : \quad & (1-\delta)(1-p(W_t,a_t))(1+r)^{-1}\Pi_V(s_{t+1}) + \beta\lambda_t = 0 \\
W_t : \quad & -p_W(W_t,a_t)\left(x_t - w_t + \tfrac{\mathbb{E}_{x_{t+1}}[\Pi(s_{t+1})|x_0,x_t]}{1+r}\right) + \eta_t(1-p(W_t,a_t)) - \tfrac{\lambda_t}{1-\delta} = 0 \\
a_{t+1} : \quad & (1-\delta)(1-p(W_t,a_t))(1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi_a(s_{t+1})|x_0,x_t\right] = \mu_t - \zeta_t
\end{aligned}$$

and the envelope conditions are

$$\begin{aligned}
V_t : \quad & \Pi_V(s_t) = -\eta_t \\
a_t : \quad & \Pi_a(s_t) = -(1-\delta)p_a(W_t,a_t)\left(x_t - w_t + (1+r)^{-1}\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]\right) \\
& \qquad + \eta_t\left(\delta U'(a_t) + (1-\delta)S_a(W_t,a_t)\right) + \mu_t(1+r)
\end{aligned}$$

Consider the optimality conditions with respect to $V(x_{t+1})$ and W_t in the optimal contract, and the envelope condition with respect to V_t to get

$$\beta^{-1}(1+r)^{-1}\eta_{t+1} - \eta_t = -\frac{p_W(W_t,a_t)}{1-p(W_t,a_t)}\left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]}{1+r}\right)$$

Now combine the first-order conditions for c_t, w_t and $V(x_{t+1})$ and the envelope condition for V_t to get

$$\eta_{t+1} = \frac{\beta(1+r)}{u'(c_t)} \tag{A.1}$$

and therefore

$$\frac{1}{u'(c_t)} - \frac{\beta(1+r)}{u'(c_{t-1})} = -\frac{p_W(W_t,a_t)}{1-p(W_t,a_t)}\left(x_t - w_t + \frac{\mathbb{E}_{x_{t+1}}\left[\Pi(s_{t+1})|x_0,x_t\right]}{1+r}\right)$$

which is equation (5).

A.3 Proof of proposition 2

First, combine the first-order condition for a_{t+1} with the envelope condition for a_t to get

$$\begin{aligned}1+r \geq \mathbb{E}_{x_{t+1}}\left[\eta_{t+1}\left(\delta U'(a_{t+1}) + (1-\delta)S_a(W_{t+1},a_{t+1})\right) + (1-\delta)(1-p(W_{t+1},a_{t+1}))(1+r)|x_t\right] \\ -\mathbb{E}_{x_{t+1}}\left[(1-\delta)p_a(W_{t+1},a_{t+1})\left(x_{t+1}-w_{t+1}+\frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}),a_{t+2},x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right]\end{aligned} \tag{A.2}$$

with equality if the borrowing constraint does not bind.

Next, combine the first-order condition for c_t and the envelope condition of the unemployed workers to get

$$U'(a_t) = (1+r)u'(c_t^u) \tag{A.3}$$

where c_t^u denotes the consumption of the unemployed worker.

Finally, we need to derive an expression for $S_a(W_{t+1},a_{t+1})$. From the envelope condition on the search problem of workers (2), we get

$$S_a(W_t,a_t) \equiv \kappa\left(v(W_t,a_t)-W_t\right)\partial_a\lambda_w(v(W_t,a_t),a_t)$$

We now derive an expression for $\partial_a\lambda_w(v(W_t,a_t),a_t)$ from the free entry condition of firms. Combining the first-order conditions and envelope conditions in the problem of new entrants gives

$$\partial_v\Pi_0(v,a_t) = -\frac{1}{u'(c_t)}$$

$$\partial_a\Pi_0(v,a_t) = 1+r$$

where c_t represents the consumption of the worker at the new job. Now consider the free entry condition

$$\lambda_f(v,a) = \frac{k}{\Pi_0(v,a)}$$

Differentiating this expression with respect to v and a gives

$$\partial_a\lambda_f(v,a) = -\lambda_f(v,a)\frac{1+r}{\Pi_0(v,a)}$$

$$\partial_v\lambda_f(v,a) = \lambda_f(v,a)\frac{1}{\Pi_0(v,a)u'(c_t)}$$

Taking the ratio gives

$$\partial_a\lambda_f(v,a) = -\partial_v\lambda_f(v,a)(1+r)u'(c_t)$$

With a constant returns to scale matching function, we can express the job finding rate as $\lambda_w(v,a) = f(\lambda_f(v,a'))$. Therefore,

$$\partial_a\lambda_w(v,a) = -\partial_v\lambda_w(v,a)(1+r)u'(c_t) \tag{A.4}$$

We can use this expression to rewrite $S_a(W_t,a_t)$ as

$$S_a(W_t,a_t) = -(1+r)u'(c_t^{ee})\kappa\left(v(W_t,a_t)-W_t\right)\partial_v\lambda_w(v(W_t,a_t),a_t)$$

where c_t^{EE} is the consumption of the worker during the first period after an EE separation. We can further simplify this term using the first-order condition of the search problem

$$\lambda_w(v_t, a_t) + \partial_v \lambda_w(v_t, a_t)\,(v_t - W_t) = 0$$

and get

$$S_a(W_t, a_t) = (1+r)u'(c_t^{ee})\kappa\lambda_w(v(W_t, a_t), a_t) = (1+r)u'(c_t^{ee})p(W_t, a_t) \tag{A.5}$$

Combine equations (A.2), (A.1), (A.3), and (A.5), to get

$$\begin{aligned} u'(c_t) \geq\; & \mathbb{E}_{x_{t+1}}\left[\beta(1+r)\left(\delta u'(c_{t+1}^{u}) + (1-\delta)p(W_{t+1}, a_{t+1})u'(c_{t+1}^{ee})\right) + (1-\delta)(1-p(W_{t+1}, a_{t+1}))u'(c_t)|x_t\right] \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\frac{p_a(W_{t+1}, a_{t+1})}{1+r}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

For the final step, rewrite the consumption growth condition (5) evaluated at $t+1$ as

$$u'(c_t) = \beta(1+r)u'(c_{t+1}) - u'(c_t)u'(c_{t+1})\frac{p_W(W_{t+1}, a_t)}{1-p(W_{t+1}, a_t)}\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}\left[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}\right]}{1+r}\right)$$

We can replace $u'(c_t)$ on the right-hand side by this expression and get

$$\begin{aligned} u'(c_t) \geq\; & \beta(1+r)\left(\delta u'(c_{t+1}^{u}) + (1-\delta)\mathbb{E}_{x_{t+1}}\left[p(W_{t+1}, a_{t+1})u'(c_{t+1}^{ee}) + (1-p(W_{t+1}, a_{t+1}))u'(c_{t+1})|x_t\right]\right) \\ & -(1-\delta)u'(c_t)\mathbb{E}_{x_{t+1}}\left[\left(u'(c_{t+1})p_W(W_{t+1}, a_t) + \frac{p_a(W_{t+1}, a_{t+1})}{1+r}\right)\left(x_{t+1} - w_{t+1} + \frac{\mathbb{E}_{x_{t+2}}[\Pi(V(x_{t+2}), a_{t+2}, x_{t+2})|x_{t+1}]}{1+r}\right)|x_t\right] \end{aligned}$$

which is equation (7).

A.4 Optimality conditions for unemployed workers

The optimality condition with respect to v_{t+1} leads to the standard optimality condition in directed search models

$$\lambda_w(v_{t+1}, a_{t+1}) = -\partial_v \lambda_w(v_{t+1}, a_{t+1})\left[v_{t+1} - U(a_{t+1})\right]$$

It states that workers equate the benefits of search in markets with higher values v_{t+1} to the cost in terms of decreased match probability.

Combining the optimality conditions with respect to c_t and a_{t+1} and the envelope condition gives

$$u'(c_t) \geq \beta\partial_a\lambda_w(v_{t+1}, a_{t+1})\left[v_{t+1} - U(a_{t+1})\right] + \beta(1-\lambda_w(v_{t+1}, a_{t+1}))(1+r)u'(c_{t+1})$$

Finally, we can use equation (A.4) and the optimality condition for v_{t+1} to get

$$u'(c_t) \geq \beta(1+r)\left[\lambda_w(v_{t+1}, a_{t+1})u'(c_{t+1}^{ue}) + (1-\lambda_w(v_{t+1}, a_{t+1}))u'(c_{t+1})\right]$$

which is a standard Euler equation.

A.5 Proof of proposition 3

The proof has two steps. First, we show that conditional on an initial value for worker assets a, the optimal allocation of the contract when assets are public information also solves the optimal contract when assets are private information. Second, we show that workers do not benefit by searching in markets indexed by assets $\tilde{a}$ when their actual asset is a. Together, these two results show that the optimal contract with public information is also optimal if assets were private information. Details coming soon.

B Data appendix

I use administrative data provided by the CASD in France between 2008 and 2019. My analysis relies on two main files:

a) the panel version of the "DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) "FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms ("Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level ("UL"), and several legal units can belong to the same firm. Since I want to measure EE separations across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the "Entreprise profilée" ("EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years and for firms with at least 3 employees (in the panel or not). I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.6}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying EE separations is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job separation. An EE separation occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as EE separations if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of EE separations with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make an EE separation experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job separations with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

When a worker separates from their previous jobs and does not make an EE separation, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. separation from private sector to public sector), I do not count it either as an EE separation nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.