The Pass-through of Productivity Shocks to Wages and the Cyclical Competition for Workers

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Abstract

Using French matched employer-employee data, I document that after positive firm-level productivity shocks, the wages of stayers rise and job-to-job transitions fall. However, after positive sectoral productivity shocks, wages rise significantly more and job-to-job transitions rise. To explain these differences, I build a model with dynamic wage contracts subject to two-sided limited commitment and imperfect information and in which sectoral productivity shocks generate cyclical competition for workers. After a positive firm-level shock, a firm increases its wages to reduce the quit rate of its workers. This increase is limited because workers are risk-averse and value insurance against shocks and because there is no increase in the cyclical competition from other firms. In contrast, after positive sectoral shocks, the cyclical competition for workers heats up and workers become more likely to switch jobs. In response, all firms increase their wages more aggressively to retain them. I find that firing costs play a new role when contracts are endogenous: by enhancing the commitment power of firms, they allow workers to receive more insurance against negative shocks.

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1 Introduction

How do firms and workers share risk when profits fluctuate? Do firms absorb productivity shocks into their profits or do they pass them on to workers through their wages? A prominent view among economists is that firms provide some form of insurance to workers through wage contracts (Knight, 1921, Baily, 1974, Azariadis, 1975). Estimates of the pass-through of productivity shocks to wages can be used to assess whether firms insure workers against these shocks or pass them through to wages. Recent evidence shows that the pass-through of firm-level productivity shocks is small (Guiso, Pistaferri and Schivardi, 2005) whereas the pass-through of sectoral productivity shocks is much higher (Carlsson, Messina and Skans, 2016). In this sense, the data suggests that firms provide relatively more insurance against firm-level shocks than against sectoral shocks.

There is essentially no existing work that simultaneously accounts for the patterns of pass-through of both firm-level and sectoral productivity shocks documented in the data. One strand of literature studies risk-sharing between firms and workers but focuses on firm-level and worker-level shocks (Balke and Lamadon, 2022). There is also a large macroeconomic literature that studies the response of wages to sectoral or aggregate productivity shocks (e.g. Moscarini and Postel-Vinay, 2013) but in models with risk-neutral workers, thus overlooking the risk-sharing problem between firms and workers.

This paper builds a model that generates these patterns of pass-through as a result of optimal contracting between firms and risk-averse workers. The key idea in this model, explained in more detail below, is that firms face a trade-off between providing insurance to workers and competing against other firms to retain workers. On one hand, providing insurance makes contracts more attractive to workers, making it easier for firms to hire them. On the other hand, passing through productivity shocks to wages helps to retain workers when they generate the most profits. Firms face additional incentives to pass through sectoral productivity shocks relative to firm-level shocks because sectoral shocks also influence the intensity of the cyclical competition for workers. I derive analytical formulas for the pass-through, which describe how firms balance the worker preference for insurance with the cyclical competition for workers. I use the model to quantify how much insurance firms provide to workers over sectoral cycles and revisit the role of firing costs when wage contracts are endogenous. Finally, I show that the contracts that firms offer to workers change significantly when workers can trade risk-free bonds.

I start by using French matched employer-employee data between 2008 and 2019 to document the pass-through of firm-level and sectoral productivity shocks to the wages of workers employed at the same firm for two consecutive years (stayers). Consistent with existing literature, I find that wages respond significantly more to sectoral shocks than to firm-level shocks. After a positive productivity shock normalized to 100%, wages increase by 4% when the shock is firm-level and by 18% when it is sectoral. Since the competition for workers is central to my model, I also measure job-to-job transition rates and find that they respond very differently to firm-level and sectoral shocks. After a 100% increase in productivity, job-to-job transitions fall by about 2 percentage points when the shock is firm-level but rise by 4 percentage points when the shock is sectoral.

To understand these facts, this paper builds an equilibrium model of the labor market with risk-averse workers and dynamic wage contracts. Workers can switch jobs but face search frictions. They receive preference shocks for changing jobs, which effectively imply that only a fraction of workers choose to search for new jobs every period. Contracts are subject to limited commitment on the side of workers and firms as in Thomas and Worrall (1988), and there is imperfect information about the worker search decision and preference shocks. Firms are heterogeneous in their permanent productivity, and experience firm-level and sectoral productivity shocks. In a baseline version of the model, I assume that workers consume their wages and I later relax this assumption by allowing workers to trade in risk free bonds, which allows them to smooth their consumption in response to shocks. My model is a quantitative version of Menzio and Shi (2010), and is closely related to Balke and Lamadon (2022).

The model captures a trade-off between retaining workers when they are most productive, and insuring them against productivity shocks. Consider first how a firm responds to firm-level shocks. If a firm provided complete insurance against such shocks by paying constant wages, its workers would leave at a constant rate. Such a firm could increase its profits by raising wages when firm productivity is high to reduce the quit rate, and lowering wages when firm productivity is low to increase it. With this strategy firms would retain workers precisely when they generate the most profits. But passing through productivity shocks to wages too much is not optimal because workers are risk-averse and they value insurance against shocks. Indeed, if one firm adopted a strategy of close to complete pass-through of its shocks, it would have to offer much higher average wages to make its offer attractive relative to an offer that has a lower pass-through and, hence, provide better insurance. Therefore, firms balance the benefits of varying wages with productivity to optimize worker retention against the benefits of providing insurance to workers against shocks so as to design contracts that both maximize profits and are attractive to workers.

In sharp contrast to firm-level shocks, sectoral shocks also affect the intensity of the competition for workers. After positive sectoral productivity shocks, all firms are more profitable and hence all of them are more eager to attract workers. This cyclical increase in competition means that if any one firm did not increase its wages, that firm would disproportionately lose its workers to poaching firms precisely when these workers would generate the most profits. Thus, because of the cyclical upswing in competition from poachers, firms raise wages more aggressively when all firms become more productive. Nonetheless, in this scenario the larger increase in wages only partly offsets the upswing in competition, so that firms lose workers at a faster rate than they would absent such sectoral shocks. In a symmetric fashion, negative sectoral shocks reduce the desire of competing firms to attract workers so firms can reduce wages significantly without causing an upswing in the quit rate of workers.

I derive a new analytical formula for the pass-through of firm-level productivity shocks to wages that yields further insights into this mechanism. Specifically, I compute the impulse response of wages to a mean-reverting productivity shock. I derive this result in continuous time using methods from Sannikov (2008), and using a novel approximation to the optimal contract that I introduce. The resulting pass-through formula shows that it is optimal to backload the wage increase in response to a positive firm-level shock, meaning that wages rise proportionately more in future periods than today relative to productivity shocks. Briefly, backloaded wages encourage workers to stay with the firm in order to benefit from these future higher wages.

This pass-through formula also shows that the tension between worker retention and insurance boils down to a ratio of a *retention elasticity* to the relative risk aversion of the worker, where the retention elasticity is the percentage point change in the worker job-to-job transition rate induced by a one percent increase in the present value of wages. When risk aversion is large, workers value insurance against shocks more and the optimal pass-through is low. In contrast, when the retention elasticity is large, increasing wages is an effective strategy to retain workers and the optimal pass-through is high. The retention elasticity is endogenous to the equilibrium, but is taken as an exogenous function of wages and shocks in the firm's problem. It turns out that this elasticity provides sufficient information for the determination of a firm's optimal policy.

A similar formula for the pass-through of sectoral productivity shocks shows when this pass-through is larger than the pass-through of firm-level shocks. First, it is larger when the firm value is larger. Intuitively, the larger is a firm's present value of profits, the stronger is the firm's desire to retain workers. Hence, this higher value induces firms to respond more aggressively to an increase in competition from outside firms following a sectoral shock. Second, the pass-through is larger when the retention elasticity increases in sectoral productivity. This second condition is especially strong for workers with currently high wages. The reason is that in normal times these workers are already paid more than nearly all poachers can offer so they are unlikely to leave. But in a boom wage offers from poachers become more attractive, these workers start searching for jobs and their retention elasticity increases greatly. Firm optimally respond by passing through a large fraction of sectoral shock to their wages.

In addition to these forces, the model features an important asymmetry in the response of wages to positive and negative shocks because firms have limited commitment. The *firm limited commitment constraint* specifies that firms will terminate matches whenever their value turns negative. Hence, the contract will imply that after a sequence of negative productivity shocks that leads the value of the firm to be zero, there must be complete pass-through of negative shocks so that the firm value does not turn negative. This stands in contrast to positive productivity shocks that only trigger smooth adjustments in wages because no such issue with termination arises. Crucially, the firm limited commitment constraint is more likely to bind after a sectoral boom when the upswing in the cyclical competition for workers has increased wages and reduced profits. In this case, the intense competition for workers has left firms more vulnerable to negative productivity shocks and workers more likely to experience sharp wage cuts.

I bring the model to the data to quantify how much insurance firms provide to workers in response to various shocks. I quantify the model using moments on firm and sectoral productivity shocks as well as labor market flows estimated in my matched employer-employee data. In the quantitative model I add a cost of terminating contracts, which I refer to as *firing costs*. This cost relaxes the firm limited commitment constraint so that firms terminate matches only if their value is more negative than the firing costs. I calibrate these firing costs using data from the International Labor Organization for France. I find that the model accounts well for the differential response of wages and job-to-job transitions to firm-level and sectoral productivity shocks, which are not targeted in the quantification. Remarkably, high productivity firms disproportionately pass through sectoral productivity shocks to workers relative to low-productivity firms. This occurs because high productivity firms have high value and hence are relatively more eager to hang on to their workers, and because these firms tend to have high-wage workers for whom the retention elasticity increases sharply following positive sectoral shocks.

I then use the quantitative model to revisit the role of firing costs when contracts are endogenous. In the baseline calibration, firing costs are substantial and as a result firms have a lot of commitment power. In particular, less than 1% of firms reach the limited commitment constraint every quarter, and as a result workers experience smooth adjustments in their wages. To isolate the role of firing costs, I compute a counterfactual in which I reduce them to a much lower level consistent with the United States. With these lower firing costs, almost 15% of firms reach the limited commitment constraint every quarter and as a result workers receive a lot less insurance against negative productivity shocks. The pass-through of firm-level productivity shocks to wages also becomes counter-cyclical: it is 15% higher in downturns than in booms. This result differs from previous work on firing costs, which emphasized their ambiguous effect on employment (Bentolila and Bertola, 1990) and their perverse effect on the reallocation of workers towards more productive firms (Hopenhayn and Rogerson, 1993).

Finally, I document in the model that wage inequality increases in downturns because of the differential dynamics of wages for incumbent workers and new hires. In response to a negative sectoral shock, the wage of incumbent workers falls slowly over time because these workers are insured through wage contracts, whereas the wage of new hires falls sharply. This result is consistent with existing work on dynamic contracts (Rudanko, 2009, Kudlyak, 2014, Basu and House, 2016). The novelty is that new hires are located at the bottom of the wage distribution whereas incumbent workers are at the top because wages grow with tenure. Thus, after a negative sectoral shock the bottom of the distribution expands sharply whereas the top remains fairly stable, and wage inequality rises. Quantitatively, I find that the cross-sectional dispersion in log wages is 12% larger in sectoral downturns than in booms.

An important assumption in my baseline model is that workers have no access to financial markets. In reality, workers have access to alternative forms of insurance, such as credit card debt, that can interact with the insurance provided by firms through wage contracts. To illustrate this point, I characterize a 2-period version of the model in which workers can trade risk-free bonds. Surprisingly, I find that risk-free bonds enhance the ability of firms to retain workers. The reason why workers are less likely to change jobs is that wages are extremely backloaded so workers forego a large part of their compensation when they switch jobs. Without risk-free bonds, backloading wages so much is not optimal because it implies a path for consumption that is also extremely backloaded, which is unattractive to workers with concave utility. With risk-free bonds, firms choose to backload wages more because they can make workers borrow and smooth consumption. In the model with risk-free bonds, a precautionary savings motive is the new force that limits the degree of backloading in wage contracts. When firms set the wage of workers, and effectively pin down borrowing, they take into account that borrowing is risky because workers might end up in the future with a lot of debt and very little income to pay for it, for example if they become unemployed. When trades in risk-free bonds are private information to workers, firms use the pass-through of productivity shocks to wages to manipulate this precautionary savings motive[1].

Related literature This paper builds on a literature studying how firms compete to attract and retain workers using dynamic wage contracts. Building on the work of Burdett and Mortensen (1998), this literature characterizes the optimal hiring and retention policy of firms and the implications for wages. In a model with random search and constant productivity, Burdett and Coles (2003) show that firms face a trade-off between preventing workers from switching jobs and smoothing their wages over time. Stevens (2004) studies a similar problem with risk-neutral workers while Shi (2009) extends this analysis to a model with risk-averse workers and directed search. One appeal of these models is that they are consistent with the well-documented fact that wages grow and job-to-job transitions fall on average during a match (e.g. Topel and Ward, 1992). Balke and Lamadon (2022) estimate a similar model with firm-level and worker-level productivity shocks to evaluate whether firms insure workers against idiosyncratic shocks through wage contracts. My paper extends this literature to the study of macroeconomic shocks, specifically productivity shocks that impact all firms in a sector. The key difference is that after sectoral shocks, firms have to respond to general-equilibrium changes in the intensity of the cyclical competition for workers. To do so, I build on the model of Menzio and Shi (2010) who consider optimal wage contracts with job-to-job transitions, firm and aggregate shocks. In their paper, they prove existence of an equilibrium that has the property of block recursivity for a broad class of models with wage contracts, including the one I build on in this paper. Relative to these papers, I derive a new analytical characterization of the optimal contract in the form of pass-through formulas, I build a quantitative version of this model with firm and sectoral shocks that I estimate using administrative data and I consider the case in which workers can trade risk-free bonds. My pass-through formulas are reminiscent of the Chetty-Baily statistic for optimal unemployment insurance (Baily, 1978, Chetty, 2006), highlighting a common structure behind these two problems.

Another branch of the literature has studied models in which firms compete for workers over the business cycle. These models have been used to explain the cyclicality of labor market flows (Menzio and Shi, 2011, Moscarini and Postel-Vinay, 2016b, Schaal, 2017, Fukui, 2020, Carrillo-Tudela, Clymo and Coles, 2021), to study the reallocation of workers towards more productive firms (Moscarini and Postel-Vinay, 2013, Coles and Mortensen, 2016, Lise and Robin, 2017, Acabbi, Alati and Mazzone, 2022) and to evaluate the relevance of a Phillips curve defined in terms of job-to-job transitions (Moscarini and

[1] In ongoing work, I extend this problem to a dynamic setting and evaluate whether the degree of wage backloading and the pass-through change significantly when workers have access to realistic asset markets.

Postel-Vinay, 2022). The model in this paper differs from most of the literature by assuming that workers are risk-averse[2]. This assumption has three implications. First, the model in this paper can be used to quantify how much insurance firms provide to workers and evaluate the role of policies, such as firing costs, that influence the risk that workers face over the cycle. Second, the model can be disciplined using numerous moments from microeconomic data, such as the pass-through of firm-level productivity shocks to wages. Third, in models with risk-neutral workers, the path of wages is not determinate unless it is assumed that all workers within a firm are paid the same wage. Another distinction between this paper and the literature is that I study sectoral shocks in the data whereas existing papers focus on aggregate shocks. In the model however, the shocks that I call sectoral are identical to the aggregate shocks studied in this literature. The reason why I focus on sectoral shocks in my empirical exercise is that in the context of a risk-sharing problems, other forces than the cyclical competition for workers might prevent firms from insuring workers against aggregate shocks[3].

There are two branches of literature on job-to-job search. In the Burdett and Mortensen (1998) approach, firms do not make counteroffers and instead preempt workers from switching jobs. In a different tradition exemplified by Postel-Vinay and Robin (2002), firms do make counteroffers. From an applied perspective, I argue that the Burdett and Mortensen approach of *no counteroffers* seems more appropriate for my setting. In particular, while it is true that, at least anecdotally, counteroffers are often made for very skilled workers, they seem to be much less prevalent for the average worker in my dataset. In setting up my model, I impose restrictions on information and technology such that it is not incentive feasible to treat workers who have received offers differently from those who have not. As a result, it is not optimal for firms to make counteroffers in my model.

In this paper, search and contracting frictions are inter-connected. Imperfect information about the worker search decisions induces firms to use wages to influence the worker job-to-job transition rate, which depends on search frictions. Early work on dynamic wage contracts focused on contracting frictions but abstracted from search frictions (e.g. Harris and Holmstrom, 1982, Holmström, 1983, Thomas and Worrall, 1988). In these models, the wage changes when the value of the outside option, such as quitting into unemployment, exceeds the value from the current job. Instead, in models with search frictions the wage changes almost continuously to influence the probability that a worker switches jobs. Recent work on dynamic wage contracts (Rudanko, 2009, 2011, Kudlyak,

[2] One recent exception is Acabbi et al. (2022) who use a model with risk-averse workers similar to mine. They focus on the persistent effects of recessions when workers have human capital.

[3] For example, aggregate risk is more difficult to insure because it cannot be diversified by investors.

2014) has also considered models with contracting frictions in which the value of unemployment depends on search frictions. However, in these models workers do not switch jobs and thus wages only adjust when the worker or the firm outside option binds. My model instead focuses on job-to-job transitions, and as such is consistent with recent evidence that job-to-job transitions are an important driver of wage growth over the cycle, even for workers who do not switch jobs (Moscarini and Postel-Vinay, 2016a, 2017, Karahan, Michaels, Pugsley, Sahin and Schuh, 2017).

Finally, this paper adds to a large empirical literature on the employment history of workers by documenting new evidence for France. Specifically, my estimates for the pass-through of firm-level and sectoral shocks are consistent with existing estimates from other countries (e.g. Guvenen, Schulhofer-Wohl, Song and Yogo, 2017, Guiso and Pistaferri, 2019). A recent focus of this literature has been to measure the cyclicality of the pass-through of firm-level shocks (Guvenen, Ozkan and Song, 2014, Chan, Salgado and Xu, 2020). I find that in France the pass-through of firm-level shocks is a-cyclical in the data, and show in my quantitative model that lowering firing costs would make the pass-through counter-cyclical. My pass-through formulas also show the critical role of the retention elasticity, a parameter that has been estimated in Kline, Petkova, Williams and Zidar (2019) and Dube, Giuliano and Leonard (2019).

Layout The paper starts in section 2 with motivating evidence on the response of wages and job-to-job mobility to firm-level and sectoral productivity shocks. Section 3 presents the model, and I characterize the optimal contract in section 4. Section 5 brings the model to the data and quantifies the risk faced by workers over sectoral cycles. In section 6, I consider an extension of the baseline model that allows workers to trade in risk-free bonds. Proofs are in the appendix.

2 Motivating evidence

I start by I documenting using matched employer-employee data that wages and job-to-job transitions respond very differently to firm-level and sectoral productivity shocks. I will use these facts as testable implications of my model.

2.1 Matched employer-employee data from France

I use administrative data from France between 2008 and 2019 to discipline my analysis. I combine annual data on firm balance sheet with a panel of worker from social security data containing 1/12th of the French labor force. Using administrative data is critical for my analysis because I estimate the response of wages and job-to-job mobility decisions at the individual level to changes in firm and sectoral productivity.

I focus on a sample of workers with relatively strong attachment to labor markets and for which I can measure job-to-job mobility accurately. Specifically, I only keep in the sample workers with permanent full time contracts, and prime age workers (25-55 years old). I focus on private sector jobs in for-profit firms with at least 3 employees. Appendix A provides more details on the sample selection and data construction and summary statistics on the population of interest. I end up with about 530,000 workers and 130,000 firms per year.

I measure labor productivity using value added per worker, controlling for the cost of capital. I measure the cost of capital as the product of tangible assets and interest rates plus depreciation rates, where interest rates are estimated from the balance sheet data and depreciation rates are estimated at the annual-sector level using national accounts data. I model labor productivity y_{jst} at firm j in sector s and at time t as

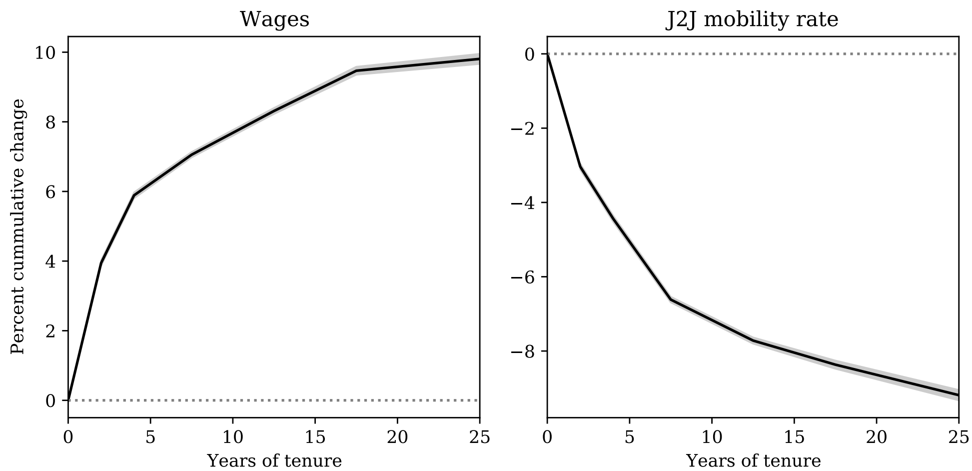
$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst}$$

where a_t is an aggregate component, z_{st} a sectoral component and x_{jst} a firm-level component. I first residualize $\log y_{jst}$ on time dummies to extract the common component and on firm age dummies to control for the life cycle of firms, which is not in the model. I then measure the sectoral component $\log z_{st}$ as the average productivity across firms within a sector and compute the firm component $\log x_{jst}$ as the residual.

I measure wages as annual labor earnings divided by the number of days worked. Given that I consider a sample of relatively stable workers, changes in hours within the day are unlikely to be large. Labor earnings are net of payroll taxes but before income taxes and they include all types of compensations, including bonuses and payment in kinds, but excludes stock options. Unlike in the U.S., medical insurance is not an important part of pay in France. I residualize the log of labor earnings on observable worker and firm characteristics, such as occupation, industry or location dummies, a gender dummy, a polynomial in experience and dummies for firm age.

I then compute the growth rate of earnings and productivity and remove outliers at the bottom and top 0.5% of the distribution each year.

Figure 1: Tenure profiles of wages and job-to-job transitions



Note: the gray areas around the line denotes the confidence intervals estimated with block bootstrap in which firms are re-sampled. Number of observations is approximately 530,000 workers per year for 12 years.

2.2 The tenure profile of wages and job-to-job transitions

I start by documenting in the data the profile of wages and job-to-job transitions with tenure within a match.

I regress residualized wages on dummies for tenures following

$$\log w_{ijst} = \alpha + \sum \delta^{\text{tenure}} + \epsilon_{ijst}$$

and use the estimates for dummy variables to measure the tenure profile of wages. Similarly, I compute the tenure profile of worker mobility by regression an indicator for job-to-job transitions on dummies for tenure.

The estimates for the tenure profiles of wages and job-to-job transitions are shown in figure 1. The results show that wages rise systematically over time for the duration of a match. At the same time, workers quit rate for another job fall. After 10 years in the match, workers are paid approximately 8% more and are 8 percentage point less likely to leave for another job than when they just matched with the firm. These results are consistent with existing literature and are often cited as evidence for dynamic wages contracts. In section 4, I show that my wage contracts are indeed consistent with this pattern.

	Wages	Job-to-job transition rate
Firm productivity shock	4.6% (0.61%)	- 1.7pp (0.89pp)
Sectoral productivity shock	18.5% (3.5%)	4.0pp (7.5pp)

Note: standard errors are shown in parenthesis and are estimated by block bootstrap in which firms are re-sampled. Number of observations is approximately 530,000 workers per year for 12 years.

Table 1: Estimated response of wages and job-to-job transitions to productivity shocks in the data

2.3 The differential response of wages and job-to-job transitions to firm and sectoral productivity shocks

I measure the response of wages and job-to-job mobility as the percent change in wages, and the percentage point change in job-to-job transition rate after a 100% increase in firm-level and sectoral productivity. These responses are estimated using standard estimators from Guiso et al. (2005).

Define the growth rate of residualized wages for worker i in firm j sector s and between year $t-1$ and t as $\Delta \log w_{ijst}$ and define the growth rate of firm and sectoral productivity as $\Delta \log x_{jst}$ and $\Delta \log z_{st}$. The response of wages to firm-level and sectoral productivity shocks are defined as

$$\theta^{w,y} = \frac{\text{Cov}(\Delta \log w_{ijst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})}{\text{Cov}(\Delta \log y_{jst}, \sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau})} \tag{1}$$

where $y \in \{x, z\}$ denotes firm-level or sectoral productivity and where $\sum_{\tau=-1}^{1} \Delta \log y_{jst+\tau}$ is the 3-year cumulative sum of productivity growth. In a model with permanent productivity shocks and static pass-through as in Guiso et al. (2005), this estimator recovers the true pass-through of productivity shocks to wages. It can be computed from a regression of wage growth on productivity growth, using the 3-year cumulative sum as an instrument for productivity growth. This instrument filters out the effect of transitory changes in productivity, which I interpret as measurement errors. In my model the pass-through is not static and shocks not permanent so these coefficients do not really measure the pass-through but I treat them as auxiliary statistics to compare my model with the data. I find it useful to report these statistics, as opposed to simple covariances, since they have been extensively documented in the literature. I measure the response of job-to-job transitions using an indicator variable J2J_{ijst} equal to 1 if worker i leaves firm j during a job-to-job transition in year t. I then recover $\theta^{\text{J2J},x}$ and $\theta^{\text{J2J},z}$ using similar estimators than (1).

Estimation results are shown in table 1 with standard errors in parentheses. I find that wages and job-to-job transitions respond very differently to firm-level and sectoral productivity shocks. Wages respond almost 4 times more to sectoral productivity shocks than to firm-level shocks, while job-to-job transitions fall after a positive firm-level shock and increase after a positive sectoral shock.

3 Model

I now present a model of business cycles that features frictional labor markets with job-to-job mobility and dynamic wage contracts subject to rich contracting frictions. This model is meant to capture the differential response of wages and worker mobility to firm-level and sectoral productivity shocks documented in the data. It is a quantitative version of Menzio and Shi (2010).

3.1 Environment

Time is discrete and runs forever at interval Δt. In the quantitative analysis in section 5 I set $\Delta t = 1$ quarter. When I characterize the contract in section 4, I take the continuous time limit $\Delta t \to 0$. The setup in continuous time can be found in appendix B.2.1.

Agents This is a small open economy model of a sector.

A continuum of ex-ante homogeneous workers can either be employed or unemployed. Workers have no access to financial markets so they consume their wage w when employed, and home production b when unemployed. I relax this assumption in section 6. They have period utility $u(w)$ and discount the future at rate β.

Firm are owned by outside investors with discount rate β. The justification for this assumption is that investors can diversify risk from firm-level and sectoral productivity shocks in financial markets since firms and sectors are atomistic. An active firm is one that is matched with a single worker. The output from that match is $\overline{x}\exp(x_t + z_t)$ with *firm specific-shocks* x_t and *aggregate shocks* z_t following mean reverting processes

$$x_t = (1-\alpha_x)x_{t-1} + \sigma_x \nu_{xt} \quad \text{and} \quad z_t = (1-\alpha_z)z_{t-1} + \sigma_z \nu_{zt}$$

where ν_{xt} and ν_{zt} are i.i.d. innovations with standard normal distribution, and $1-\alpha_x$ and $1-\alpha_z$ parameterize the persistence of these shocks. Firm fixed productivity $\overline{x}$ is drawn at the start of the match, independently across firms. This productivity stays constant over time and lasts for the length of the match. Thus, each firm is subject to one common aggregate shock z_t and has two firm-specific shocks $(x_t, \overline{x})$.

In each period workers receive preference shocks for moving ξ_t which increments utility only if the worker moves to any new job in that period. These shocks are i.i.d. over time and across workers with distribution $\mathcal{N}(0, \sigma_\xi^2)$. The motivation for these shocks is that they capture non-monetary reasons why workers change jobs. In practice, they will help the quantitative model match the large number of job transitions with negative wage changes. Briefly, when workers receive large enough positive shocks for moving, they can increase their utility by accepting jobs with lower wages. In contrast, when workers receive sufficiently large negative shocks they will not search because no firm is willing to offer a sufficiently high wage to compensate them for the cost of moving.

Timing Each period, the sequence of events is as follows

a) Firm-level shocks x_t and sectoral productivity shocks z_t are realized

b) Firms produce and pay current wages; workers consume

c) Job mobility phase: preference shocks ξ_t for moving are realized; employed and unemployed workers search for jobs; firms post vacancies; new matches are formed and new contracts are signed

d) Quits and exogenous separations into unemployment occur

Directed search There is a continuum of labor markets indexed by the promised value to a worker denoted v. Every period, workers choose in which labor market v to apply, and firms choose where to post vacancies. Both employed and unemployed workers search in the same labor markets. Firms post vacancies in these labor markets and, only after they match, learn about their productivity x and $\overline{x}$.

Denote $\phi_u(v, z)$ and $\phi_e(v, z)$ the mass of unemployed and employed workers searching for a job and denote $\phi_f(v, z)$ the mass of vacancies posted by firms. Let κ denote the search intensity of employed workers relative to unemployed workers. In the notation that I use, I rely on the result that only current sectoral productivity z is an aggregate state in this economy so that policies need not be indexed by time or distributions. This is a consequence of Block Recursivity (Menzio and Shi, 2010, 2011).

In each labor market, a constant returns to scale matching function $\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)$ turns workers searching for a job and vacancies into matches. Define the job finding rate $\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that an unemployed worker finds a job, and the vacancy filling rate $\tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f)$ as the probability that a vacancy finds a worker. These probabilities are defined in the usual way as

$$\tilde{\lambda}_w(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_u + \kappa\phi_e}, \quad \tilde{\lambda}_f(\phi_u + \kappa\phi_e, \phi_f) \equiv \frac{\mathcal{M}(\phi_u + \kappa\phi_e, \phi_f)}{\phi_f}$$

Since these matching probabilities will depend on v and z in equilibrium, we can write them in short-hand notation as

$$\lambda_w(v,z) \equiv \tilde{\lambda}_w(\phi_u(v,z) + \kappa\phi_e(v,z), \phi_f(v,z)), \quad \lambda_f(v,z) \equiv \tilde{\lambda}_f(\phi_u(v,z) + \kappa\phi_e(v,z), \phi_f(v,z))$$

I assume that the probability that a worker finds a job is at most 1, in that $\lambda_w(v,z) \leq 1$.

In equilibrium there will be an upper bound $\overline{v}$ on the set of active labor markets. For $v > \overline{v}$, the job finding rate is not defined because no firm post vacancies there. I extend this function by setting it to 0 for these values above $\overline{v}$. This convention will be useful when I describe the choice of employed workers with preference shocks. In particular, workers who receive extremely low preference shocks will choose not to search for a new job because there is virtually no firm offering a sufficiently high wage for them to want to switch jobs. Instead of writing explicitly that these workers do not search, I write that they search in labor markets with value $v > \overline{v}$ where the job finding rate is 0.

Unemployed workers Unemployed workers consume their endowment b and choose in which labor market v to search, for working in the next period. Given the job finding probability, $\lambda_w(v,z)$, the value of unemployed workers satisfies

$$U(z_t) = u(b) + \beta \max_v \left[\lambda_w(v,z_t)v + (1 - \lambda_w(v,z_t))\mathbb{E}_{z_{t+1}}\left[U(z_{t+1})|z_t\right]\right]$$

In choosing in which labor market v to search, workers face the following trade-off: searching in a high-v labor market brings a higher value v conditional on a match, but it will turn out that these matches occur with lower probability because $\lambda_w(v,z_t)$ will decrease with the value v in equilibrium. Here all unemployed workers search in the same labor market, denoted $v_u(z_t)$, that depends only on their common state z_t.

Employed workers Employer workers also search for new jobs. Their preference shocks for moving ξ_t are realized and then they decide which market v to search in. With probability $\kappa\lambda_w(v,z_t)$, they find a new match in market v. Existing matches break up and workers separate into unemployment for two reasons. First, with exogenous probability δ a match is dissolved. Second, workers can quit voluntarily. Note that when productivity is sufficiently low, firms might want to induce a separation by reducing the wage sufficiently to make workers quit.

Contracts When firms and workers are first matched, they sign wage contracts. The contract is subject to limited commitment by workers and firms. In particular, workers cannot commit to turn down a job offer when they receive one, and cannot commit not to quit into unemployment. Contracts are also limited by private information in that a worker search decision and preference shocks for moving are both private information to that worker. This private information leads to a moral hazard problem with both a hidden action v and a hidden state ξ. Productivity shocks $(\overline{x}, x_t, z_t)$ are public information.

Firms also have limited commitment. Intuitively, this limited commitment captures the inability of a firm to commit to a contract that after some histories of shocks it would like to renege on. To model this limited commitment, I assume that if, after signing a contract, a firm chooses to walk away from it, by effectively firing the worker, it must pay a cost Φ to do so. Technically, this ability to walk away from a contract implies that in an incentive compatible contract, after any history the continuation value for the firm must be greater than $-\Phi$. Clearly, as this cost increases from 0 to some larger value, the set of contracts the firm can credibly commit to increases. I think of Φ as capturing a degree of commitment power. When $\Phi = 0$, firms have no commitment at all in that they can walk away from the match at any history for which the continuation value is negative. When $\Phi = \infty$, firms have full commitment in that they cannot walk away from a match no matter how negative the continuation value becomes. In the quantitative model, I will calibrate the commitment power of firms using estimates of firing costs. The idea behind this calibration is that if a firm decides to walk away from a deal, it effectively has to fire the worker and must therefore pay the firing costs.

As mentioned in the introduction, I follow the Burdett and Mortensen (1998) approach in which firms do not make counteroffers. I now briefly describe two assumptions such that it is not incentive feasible for firms to make counteroffers to workers. First, job offers are private information to workers and poaching firms and workers cannot provide any (contractible) evidence to their current employer. Second, workers receive counteroffers at the end of a given period when their current employers are closed, and must accept this offer before the beginning of next period when their current employers open. This assumption on timing ensures that workers cannot renegotiate with their current employers using the new job offers as outside option since it would be too late to accept the new job. Taken together, these assumptions imply that if a firm wanted to make a counteroffer to one of its employee who has claimed to receive an offer, other employees who did not receive that offer would pretend that they did in order to benefit from the same deal.

Therefore, firms cannot make counteroffers in any incentive compatible contract. In the appendix, I discuss a variant of the island model of Lucas and Prescott (1974) in which the physical transit times between islands justifies these assumptions.

Following previous work on dynamic contracts with hidden information, I write the contract recursively in terms of promised values and continuation values instead of histories of shocks. I also define incentive compatibility using *temporary incentive constraints* instead of constraints that depend on the entire history of shocks and reports[4] (Green 1987, Atkeson and Lucas 1992). A critical assumption that I use in doing so is that preference shocks ξ_t are i.i.d. over time. This implies that the continuation value of workers over contracts only depends on the reported preference shock, and not the realized shock.

Denote V_t the promised value of a worker at the start of the period and $s_t = (\overline{x}, x_t, z_t)$ the state of productivity, where $\overline{x}$ is the fixed firm productivity, x_t is the firm-level productivity shock and z_t is the sectoral productivity shock. Note that at time t, firms have different firm-specific productivity $(\overline{x}, x_t)$ but share the same sectoral productivity z_t. The state of a match at the beginning of the period are the worker promised value V_t as well as the current productivity s_t. After wages are paid and consumed, workers draw a preference shock ξ_t that becomes a part of the state at this point.

The components of the contract at time t are the wage paid today and a set of continuation values for each state tomorrow. The wage w_t is a function of the worker promised value V_t and productivity state s_t whereas the continuation values $V_{t+1}(s_{t+1}, \xi_t)$ are also functions of future productivity and of the realized preference shock ξ_t today. Formally, the wage and continuation values are represented by two functions

$$w_t\left(V_t, s_t\right) \quad \text{and} \quad V_{t+1}(s_{t+1}, \xi_t; V_t, s_t)$$

A *contract* is a collection of these functions for all t.

Given a contract, the worker chooses a search strategy and a quit strategy to maximize the present value of utility. Both the search strategy $v_t(\xi_t)$ and quit strategy $q_t(\xi_t)$ depend on preference shocks ξ_t because workers make these decisions after they observe ξ_t.

The value of a worker in state s_t given a contract and strategies $v_t(\xi_t), q_t(\xi_t)$ satisfies

$$V_t(s_t) = u(w_t) + \beta \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v_t(\xi_t), z_t)\left(v_t(\xi_t) + \xi_t\right) + (1 - \kappa\lambda_w(v_t(\xi_t), z_t))W_{t+1}(\xi_t)\right] \quad (2)$$

[4]In order to keep notations simple, I also abstract from randomized contracts.

where $W_{t+1}(\xi_t)$ is the continuation value for workers who did not find another job

$$W_{t+1}(\xi_t) = (\delta + (1-\delta)q_t(\xi_t))\,\mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] \\ +(1-\delta)(1-q_t(\xi_t))\mathbb{E}_{x_{t+1},z_{t+1}}\left[V_{t+1}(s_{t+1},\xi_t)|x_t,z_t\right] \tag{3}$$

In (2), the first term is the utility from consuming wages today. The second term depends on the probability that a worker finds another job $\kappa\lambda_w(v_t(\xi_t),z_t)$ and on the value that the worker receives if a job-to-job transition occurs $v_t(\xi_t)+\xi_t$, where we recall that a worker who prefers not to search selects an inactive labor market. In this expression, expectations over ξ_t are not conditioned on past shocks because preference shocks are i.i.d. A worker who does not find a new job gets a continuation value for non-matched workers of $W_{t+1}(\xi_t)$, which at this point can depend on the realization of the current preference shock ξ_t. This term is defined in (3). It is the sum of the value of unemployment if the worker decides to quit or if the match is dissolved exogenously, and the continuation value at the same job for the following period $V_{t+1}(s_{t+1},\xi_t)$.

Denote the worker's report of preference shock at t by $\hat{\xi}_t$. A contract is temporary incentive compatible if, for any realization for the preference shock ξ_t and productivity s_t, and given continuation values $\left\{V_{t+1}(s_{t+1},\hat{\xi}_t)\right\}$, it is optimal for the worker to report the preference shock ξ truthfully and search v and quit q decisions are optimal

$$\xi_t \text{ solves } \max_{\hat{v},\hat{\xi},\hat{q}} \kappa\lambda_w(\hat{v},z_t)\left(\hat{v}+\xi_t\right) + (1-\kappa\lambda_w(\hat{v},z_t))W_{t+1}(\hat{\xi}) \tag{4}$$

where the continuation value $W_{t+1}(\hat{\xi})$ is defined in (3). Remember that that the preference shock ξ_t is private information to workers, so the state-dependent continuation values $\left\{V_{t+1}(s_{t+1},\hat{\xi})\right\}$, and hence $W_{t+1}(\hat{\xi})$, can only depend on the report of the worker $\hat{\xi}$ and not on the realized preference shock.

Fixing the continuation value for non-matched workers $W_{t+1}(\hat{\xi})$, a worker with high preference shock ξ_t will search in labor markets with a lower value $\hat{v}$ and a higher job finding rate $\lambda_w(\hat{v},z_t)$ because this worker really values getting a new job. Conversely, a worker with low preference shock ξ_t will search in markets with a high value $\hat{v}$ and a low job finding rate. A worker who draws a sufficiently low value for ξ_t will choose not to search since there does not exist an active market offering a sufficiently high value for the worker to be willing to move.

One might postulate that firms want to promise relatively high future wages to workers with high preference shock ξ_t in order to retain them, and relatively low future wages to workers with negative preference shocks ξ_t because they are unlikely to leave anyway. But, this strategy would not be incentive compatible. Indeed, from (4), it is immediate that the continuation value $W_{t+1}(\hat{\xi})$ cannot depend on the reported preference shock $\hat{\xi}$. Otherwise, any worker would benefit from reporting the preference shock with the highest value. I now argue that if $W_{t+1}(\hat{\xi})$ is independent of $\hat{\xi}$, then so are the state-dependent continuation values $\{V_{t+1}(s_{t+1}, \hat{\xi})\}$. Holding fixed $W_{t+1}(\hat{\xi})$, allowing the continuation values $V_{t+1}(s_{t+1}, \hat{\xi})$ to vary with the preference shock ξ_t will never be optimal for firms because workers are risk-averse and this variation does not relax the incentive constraint for v, which by construction depends only on $W_{t+1}(\hat{\xi})$. Therefore, when I state the optimal contracting problem below I write the state-dependent continuation values $\{V_{t+1}(s_{t+1})\}$ as a function of future productivity states s_{t+1} only, and not of preference shocks ξ_t. Furthermore, since the quit decision only depends on these continuation values and on the value of unemployment, I will also write the worker quit policy q as independent of the current preference shock. Note, however, that the worker search decision $v(\xi_t)$ still depends on the realization of the preference shock because the value that a worker gets by changing job increases in it.

We are now ready to write the optimal contracting problem. Denote $\Pi(V, s_t)$ the present value of profits for a firm matched with a worker with promised value V and when productivity is currently $s_t = (\overline{x}, x_t, z_t)$. Taking as given the value of unemployment $U(z)$ and the job finding rate $\lambda_w(v, z)$, the value of a firm satisfies

$$\begin{aligned}\Pi(V, s_t) = \quad & \max_{w, V(s_{t+1})} \; \overline{x}\exp(x_t + z_t) - w \\ & + \beta\left(1 - \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(\xi_t), z_t)\right]\right)(1-\delta)(1-q)\mathbb{E}_{x_{t+1}, z_{t+1}}\left[\Pi(V(s_{t+1}), s_{t+1})|x_t, z_t\right]\end{aligned}$$

subject to

$$\begin{aligned}
&\text{(PK)}: && V \leq u(w) + \beta\left(W_{t+1} + \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(\xi_t), z_t)\left(v(\xi_t) + \xi_t - W_{t+1}\right)\right]\right) \\
&\text{(IC-v)}: && v(\xi_t) \in \arg\max_{\hat{v}} \; \lambda_w(\hat{v}, z_t)\left(\hat{v} + \xi_t - W_{t+1}\right) \\
&\text{(IC-q)}: && q = 1 \quad \text{if} \quad \mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] \geq \mathbb{E}_{x_{t+1}, z_{t+1}}\left[V(s_{t+1})|x_t, z_t\right] \\
&\text{(PC-F)}: && \Pi(V(s_{t+1}), s_{t+1}) \geq -\Phi
\end{aligned}$$

where $W_{t+1} = (\delta + (1-\delta)q)\,\mathbb{E}_{z_{t+1}}[U(z_{t+1})|z_t] + (1-\delta)(1-q)\mathbb{E}_{x_{t+1}, z_{t+1}}\left[V(s_{t+1})|x_t, z_t\right]$.

The firm chooses the current wage w and state-dependent continuation values $V(s_{t+1})$ to maximize the present value of profits, where $(1 - \mathbb{E}_{\xi_t}[\kappa\lambda_w(v(\xi_t), z_t)])(1-\delta)(1-q)$ is the probability that the worker remains within the current match next period, often called the *retention probability*. When the firm computes this retention probability, it takes expectation over the different values that preference shocks ξ_t can take. By changing continuation values in the future, $V(s_{t+1})$, firms influence not only future profits but also the quit and search decisions of workers q and v today and therefore the retention probability. Section 4 characterizes this trade-off between future profits and worker retention, and how it changes with firm-level and sectoral productivity shocks.

The first constraint (PK) is the *promise keeping constraint*, stating that the value the worker gets from the contract either through wage today or future values must deliver at least the promised value V. The second constraint (IC-v) is the *incentive compatibility constraint for the search* strategy v of the worker. It defines the search strategy that a worker chooses as a function of the preference shock ξ_t and the continuation value W_{t+1}. The third constraint (IC-q) is the *incentive compatibility constraint for quits* into unemployment. It states that a worker will quit if the expected value of unemployment exceeds the continuation value from the current match. The last constraint (PC-F) is the *participation constraint of the firm*, which states that the firm value cannot go below the cost Φ after any history. If this constraint was violated, firms would rather walk away from the match than continue and deliver the value V to the worker.

Value of a match Consider the value of a firm when it is just matched with a worker in market v when sectoral productivity is z_t. At this point, the firm has not yet drawn its idiosyncratic shocks $(\overline{x}, x_t)$. Denote $\Pi_0(v, z_t)$ the value from this match. It solves

$$\begin{aligned} \Pi_0(v, z_t) = \quad & \max_{V(s_{t+1})} \mathbb{E}_{\overline{x}, x_{t+1}, z_{t+1}} \left[\Pi(V(s_{t+1}), s_{t+1}) | z_t \right] \\ \text{s.t.} \quad & \mathbb{E}_{\overline{x}, x_{t+1}, z_{t+1}} \left[V(s_{t+1}) | z_t \right] = v \end{aligned}$$

where the initial value of firm productivity x_{t+1} is drawn from $x_t \sim \mathcal{N}(0, \sigma_x^2)$ and the fixed firm productivity is drawn from $\log \overline{x} \sim \mathcal{N}(0, \sigma_{\overline{x}}^2)$.

Free entry Firms are subject to a free entry condition. They have to pay a unit cost for posting a vacancy k. The free entry condition is

$$-k + \lambda_f(v, z_t) \beta \Pi_0(v, z_t) \leq 0 \tag{5}$$

with equality for each active market v. Note that the firm value $\Pi(V, s_t)$ is decreasing in the worker value V, since raising V tightens the promise keeping constraint. Thus, the value of a match is lower in markets with a high worker value v than in markets with low worker value, that is $\Pi_0(v, z_t)$ decreases in v. Consequently, the free entry condition (5) implies that the vacancy filling rate $\lambda_f(v, z)$ is increasing in the worker value v in equilibrium. Intuitively, this condition requires that firms must be indifferent between posting vacancies in markets with a high worker value v, and therefore low match value and high vacancy filling rate, and in markets with a low worker value. Finally, this results implies that the job finding rate $\lambda_w(v, z)$ is decreasing in v from the matching function. Intuitively, in markets with low worker v, there are relatively more firms posting vacancies than workers search for a job so the job finding rate is high.

3.2 Definition of an equilibrium

A recursive equilibrium is a set of value functions, policies and matching rates for each labor market v such that i) the firm and worker strategies satisfy the optimal contract, ii) the free entry condition is satisfied and iii) the job finding and vacancy filling rates are consistent with the matching function.

Denote the probability density function of the distribution of unemployed and employed workers by $\{\psi_t^u, \psi_t(V, \overline{x}, x)\}$. The resulting equilibrium outcome path is a sequence of distributions such that, given the policies, the laws of motion of distributions, defined in appendix B.1, are satisfied. These distributions are functions of time, not just of sectoral productivity, because they depend on the entire history of shocks.

4 Characterizing the optimal contract

Before bringing this model to the data, I characterize optimal wage contracts. My main focus is on understanding the effects of job-to-job mobility on wage growth and on the pass-through of firm-level and sectoral productivity shocks to wages. For this reason I strip down the model to a simpler version in order to obtain clean analytical formulas. Specifically, I assume that firms have full commitment power ($\Phi \to \infty$) and have the same fixed heterogeneity $\overline{x} = 1$ and that workers cannot quit or fall into unemployment ($\delta = q = 0$) and have no preference shocks ($\sigma_\xi = 0$).

In order to derive analytical solutions, I use a continuous time formulation of the problem ($\Delta t \to 0$). Firm-level and sectoral productivity follow

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \quad \text{and} \quad dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}$$

which are the continuous time analogue of the AR-1 process of the quantitative model.

The optimal contracting problem becomes[5]

$$\Pi(V_0, x_0, z_0) = \max_{w, \Delta_x, \Delta_z} \quad \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \int_0^t \kappa\lambda_w(v_s, z_s)ds\right)(\exp(x_t + z_t) - w_t)dt\right]$$

[5]The optimal contract in continuous time is derived from primitives in appendix B.2.1.

subject to

$$\begin{aligned}
&\text{(PK)}: \quad dV_t = (rV_t - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t))\, dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\
&\text{(IC-v)}: \quad v_t = v(V_t, z_t)
\end{aligned}$$

where $r = 1/\beta - 1$ is the discount rate of firms and workers. As before, the search decision $v(V_t, z_t)$ solves the static problem

$$v(V_t, z_t) \in \arg\max_v \ \lambda_w(v, z_t)(v - V_t) \tag{6}$$

In discrete time, firms choose the variable $V(x_{t+1}, z_{t+1})$, which describes the state-dependent continuation value of workers at $t+1$. Analogously, in continuous time firms choose the variables Δ_x, Δ_z, which measure how the worker value changes in response to firm-level and sectoral productivity shocks. The choice variables Δ_x, Δ_z are critical for my analysis because they measure the pass-through of productivity shocks to the value of workers. I characterize them in propositions 2 and 3. The promise keeping constraint (PK) describes the law of motion of the worker's value dV_t under the contract, and is the continuous time limit of the promise keeping constraint in discrete time. The HJB corresponding to this problem is given in appendix B.2.3.

Solution method to derive analytical formulas This optimal contract is difficult to characterize for two reasons: the problem is non-linear and dynamic. I now explain how I circumvent these difficulties to obtain an analytical characterization of the problem.

The first challenge with this contracting problem is that policies are non-linear. For example, from the HJB, the optimal value pass-through $\Delta_x(V, x, z)$ satisfies the optimality condition $\Delta_x(V, x, z) = -\Pi_{Vx}(V, x, z)/\Pi_{VV}(V, x, z)$. The term $-\Pi_{Vx}(V, x, z)$ measures the benefit of increasing the worker value V when firm productivity x increases, and thus captures the benefit of increasing the pass-through. In contrast, the term $\Pi_{VV}(V, x, z)$ captures the cost of varying the worker value because the worker is risk-averse, and thus captures the benefits of providing insurance to workers. Because these terms do not admit closed-form expressions, the optimal pass-through $\Delta_x(V, x, z)$ also does not.

To circumvent this difficulty, I introduce a novel approximation of the optimal contract. I characterize the contract to first-order in the search efficiency κ of employed workers around $\kappa = 0$. This is an approximation in a parameter κ that disciplines the ability of workers to change jobs. When $\kappa = 0$, there is no job-to-job mobility and the optimal contract features constant wage. To first order in κ, the optimal pass-through $\Delta_x(V, x, z)$ can be computed analytically as I show in proposition 2. In a sense, this is an approximation of the optimal contract in the contracting friction. When $\kappa = 0$, there is no friction and we get perfect insurance. As κ increases, the optimal contract places more weight on incentives arising from the cyclicality competition for workers relative to insurance.

The second challenge with this contracting problem is that policies are dynamic. Specifically, the size of the optimal pass-through depends on the persistence of productivity shocks. Furthermore, the pass-through itself is dynamic in that wages tend to respond to productivity shocks with delay as I show in sections 4.3. As a result, it is not always possible to obtain closed-form expressions for the pass-through.

To circumvent this difficulty, I characterize the contract in terms of differential equations. For example, I will write that the present value of output, denoted $g(x,z)$, satisfies $rg(x,z) = \exp(x+z) + \mathcal{D}g(x,z)$, where the differential operator $\mathcal{D}$ is defined as

$$\mathcal{D}g(x,z) \equiv -\alpha_x x g_x(x,z) - \alpha_z z g_z(x,z) + \frac{\sigma_x^2}{2} g_{xx}(x,z) + \frac{\sigma_z^2}{2} g_{zz}(x,z)$$

The expression for $g(x,z)$ is the equivalent of a Bellman equation in discrete time and the term $\mathcal{D}g(x,z)$ should be interpreted as an adjustment due to the mean-reversion and volatility of shocks. The operator $\mathcal{D}$ is useful because it allows me to characterize the contract under general conditions. I will use this notation to describe the main determinants of the firm value $\Pi(V,x,z)$ in lemma 1, and of the optimal value pass-through Δ_x and Δ_z in propositions 2 and 3. When feasible, I will also provide specific assumptions under which these equations can be solved in closed form. For example, if firm and sectoral productivity follow random walks ($\rho_x = \rho_z = 1$), the present value of output is $g(x,z) = \exp(x+z)/(r - (\sigma_x^2 + \sigma_z^2)/2)$. In appendix B.3, I derive many of the results from this section in a 2-period model where this issue of dynamic responses does not arise.

4.1 The firm value $\Pi(V,x,z)$

I first derive an expression for the firm value $\Pi(V,x,z)$ under the optimal contract. I use it to understand how worker mobility affects firm profits, and how the conflicting incentives of firms and workers lead to a moral hazard problem.

Lemma 1. *To first-order in κ, the firm value is*

$$\Pi(V,x,z) = g(x,z) - \frac{w(V)}{r} - \kappa \ell(V,x,z)$$

where $g(x,z) = (\exp(x+z) + \mathcal{D}g(x,z))\,/r$ is the present value of output and $w(V) = u^{-1}(rV)$ is the optimal wage when $\kappa = 0$. The term $\ell(V,x,z)$ represents the cost of job-to-job mobility for *firms and satisfies*

$$r\ell(V,x,z) = \underbrace{\lambda_w(v(V,z),z)}_{\text{Prob. of J2J transition}} \left[\underbrace{g(x,z) - \frac{w(V)}{r}}_{\text{Lost profits}} - \underbrace{\frac{v(V,z)-V}{u'(w(V))}}_{\text{Gains from job mobility}} \right] + \mathcal{D}\ell(V,x,z)$$

Proof. See appendix B.2.3. □

Lemma 1 characterizes the firm value for each worker promised value V and each productivity state x,z. The firm value $\Pi(V,x,z)$ depends on the present value of output, $g(x,z)$, and on the cost of wages paid to the worker $w(V)/r$.

The value of the firm also depends on the cost of job-to-job mobility, denoted $\ell(V,x,z)$. When a worker switches job, the firm looses a stream of profits. In expectation, the cost of loosing a worker for firms is $\lambda_w(v(V,z),z)\,(g(x,z) - w(V)/r)$. However, firms might also benefit from job-to-job mobility to some extent because it makes it easier to hire workers in the first place. In particular, the second term in $\ell(V,x,z)$ shows that firms benefit from job-to-job mobility because it allows them to provide workers with the same promised value V and pay them less. When workers switch jobs, they incur a utility gain of $v(V,z) - V$, which can be translated into units of wages using $u'(w(V))$. The firm can then provide some of the promised value V to workers through future job transitions, instead of future wages. For example, consulting jobs are often seen as a stepping stone for a career in management. As a result, consulting firms might find it easier to hire workers in the first place because they offer a high option value of future employment opportunities.

The moral hazard problem arises because there is an asymmetry in how much firms and workers value job-to-job mobility. To see why, it is useful to compare the optimal search decision of workers with hidden information from (6)

$$\lambda_w(v,z_t) + \frac{\partial \lambda_w(v,z_t)}{\partial v}\,(v - V_t) = 0$$

to the search decision with full information when firms control the search decision

$$\lambda_w(v^{\text{FI}},z_t) + \frac{\partial \lambda_w(v^{\text{FI}},z_t)}{\partial v^{\text{FI}}}\left(v^{\text{FI}} - \left[V_t^{\text{FI}} + \Pi(V_t^{\text{FI}},x,z)u'(w_t^{\text{FI}})\right]\right) = 0$$

The first term in these expressions measures the expected benefit of searching in markets with higher values v: workers receive a higher value conditional on matching. The second term measures the cost: the worker is less likely to match. In the case with full information, firms take into account both the worker and firm values when they compute the surplus from job-to-job transitions $v^{\text{FI}} - \left[V_t^{\text{FI}} + \Pi(V_t^{\text{FI}},x,z)u'(w_t^{\text{FI}})\right]$. In contrast, with hidden information workers only take into account their own gain $v - V_t$ when they compute this surplus. When the firm value is positive $\Pi(V, x, z) > 0$, workers overstate the surplus form a job-to-job transition relative to firms. Intuitively, both the worker and the firm want the worker to find a better job, but the worker wants it more than firms. This asymmetry is the reason why firms use wage contracts to influence the search decision of workers by making wages vary over time and in response to productivity shocks.

4.2 The path of wages

I now derive an expression showing how wages change over time and in response to shocks. I use this expression to describe the tenure profile of wages in this section, and the pass-through of productivity shocks in sections 4.3 and 4.4.

Before characterizing the path of wages, I introduce an important piece of notation. Define the *retention elasticity* $\epsilon(V, x, z)$ as the percentage point change in the retention probability induced by a 1% increase in the present value of wages, that is

$$\epsilon(V,x,z) \equiv \underbrace{\frac{\partial\left(1-\kappa\lambda_w(v(V,z),z)\right)}{\partial v(V,z)}}_{\text{Change in retention probability}} \times \underbrace{\frac{\partial v(V,z)}{\partial V}}_{\substack{\text{Change in}\\ \text{search decision}}} \times \underbrace{\frac{w(V,x,z)u'(w(V,x,z))}{r+\kappa\lambda(v(V,z),z)}}_{\substack{\text{Change in worker value after}\\ \text{1\% increase in PV of wages}}} \geq 0 \quad (7)$$

It turns out that this elasticity is the critical determinant of the retention strategy of firms because it measures the extent to which firms can influence the worker quit rate. It is positive in equilibrium: paying workers more makes them search in labor markets with a higher value v and a lower job finding rate $\lambda_w(v, z)$.

I now derive the path of wages.

Proposition 1. *The path of wages satisfies*

$$dw_t = (r + \kappa\lambda_w(v(V_t, z_t), z_t))\, \Pi(V_t, x_t, z_t)\, \frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)} dt + 0 \times dB_{xt} + 0 \times dB_{zt}$$

where $\gamma(w) \equiv -wu''(w)/u'(w)$ is the coefficient of relative risk aversion.

Proof. See appendix B.2.3. This condition is derived by combining the optimality condition with respect to the wage w_t with the envelope condition. Note that this result does not require the approximation $\kappa \to 0$, and does not use the operator $\mathcal{D}$. □

Proposition 1 is a generalization of theorem 1 in Burdett and Coles (2003) and lemma 3.2 in Shi (2009) to an environment with productivity shocks, and it is the continuous time analogue of proposition 2 in Balke and Lamadon (2022) with sectoral shocks.

The intuitions behind this equation are well established. If the firm value $\Pi(V_t, x_t, z_t)$ is positive, it is optimal for firms to backload wages to induce workers to stay. Backloading wages helps to retain workers because it makes them search in labor markets with a relatively high value and a low job finding rate. Because wages are backloaded, they grow over time and therefore $dw_t > 0$. When the elasticity of inter-temporal substitution $1/\gamma(w_t)$ is low, wages are less backloaded because workers dislike changes in consumption over time. When the retention elasticity $\epsilon(V_t, x_t, z_t)$ is large, wages are more backloaded because this strategy induces a large change in the quit rate. Wages become constant when the retention elasticity $\epsilon(V_t, x_t, z_t)$ reaches 0 because at this point the job to job transition rate of workers does not respond to changes in wages anymore. Wages also become constant when the present value of profits $\Pi(V, x, z)$ is 0 because at this point the firm is indifferent between retaining workers or letting them go.

The tenure profile of wages and job mobility An implication of proposition 1 is that wages increase and job-to-job transitions fall with tenure, consistent with the evidence documented in figure 1. To see this, note that the wage of new hires grows over time if the firm value $\Pi(V, x, z)$ and the retention elasticity $\epsilon(V_t, x_t, z_t)$ are positive. From the free entry condition,

$$-k + \lambda_f(v, z_t)\mathbb{E}_{x_t}\left[\Pi(v, x_t, z_t)\right] = 0$$

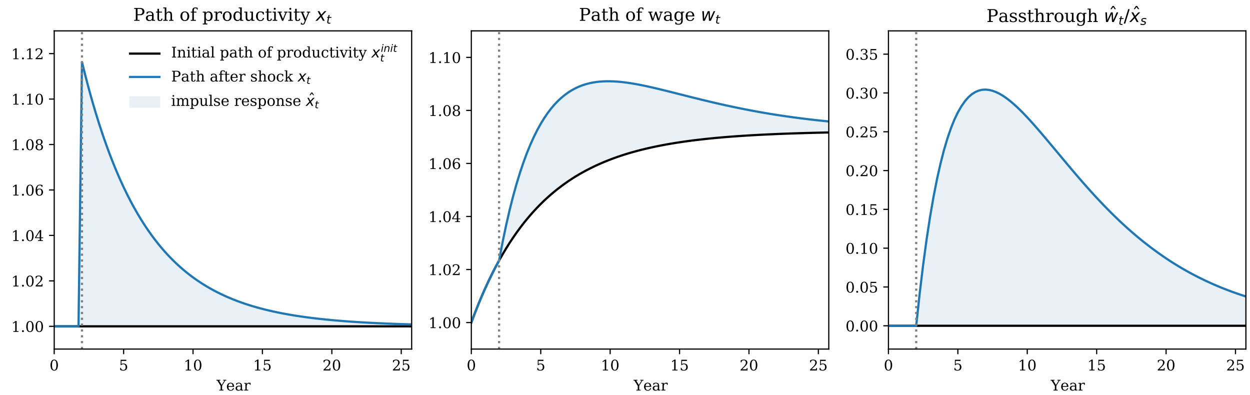
we can see that the firm value is positive in expectation since $\mathbb{E}_x\left[\Pi(v, x_t, z_t)\right] = k/\lambda_f(v, z_t) > 0$. Since the retention elasticity is also positive, the wage of new hire grows on average. Intuitively, since firms make zero profit in expectation from posting a vacancy, they must make strictly positive profits ex-post from a match to compensate for the cost of posting a vacancy k. This means that workers start a match with a relatively low wage. Their wage then increases over time as firms try to retain them by backloading wages, as shown in proposition 1. Besides, the job-to-job transition rate $\kappa\lambda_w(v(V, z), z)$ falls over time because workers choose to search in labor markets with higher value as they receive increasingly high wages.

4.3 The pass-through of firm-level shocks

I now characterize the pass-through to firm-level productivity shocks to the wage and value of stayers. I first derive a formula for the impulse response of wages w_t following a 1% increase in firm productivity and then derive the response of the worker value V_t.

The starting point to compute the pass-through to wages is proposition 1. Clearly, wages do not respond on impact to a change in productivity because the coefficient on the

Figure 2: Impulse responses from firm productivity shock



Brownian motion B_{xt} is zero. However, changes in productivity alter the path of wages through a change in the firm value $\Pi(V, x, z)$ in the drift (the retention elasticity in the drift is constant conditional on w from equation (7)). An increase in productivity x raises the firm value $\Pi(V, x, z)$ and thus increases the growth rate of wages going forward.

To solve for the response of wages to a productivity shock explicitly, I rely on the approximation as $\kappa \to 0$. In this case, the path of wages from proposition 1 becomes

$$dw_t = (rg(x_t, z_t) - w_t) \frac{\epsilon(V_t, x_t, z_t)}{\gamma(w_t)} dt$$

where the retention elasticity is defined in equation (7).

Consider a worker with some initial path for productivity and wages $\{x_t^{\text{init}}, w_t^{\text{init}}\}$. I simulate a small unanticipated shock to productivity $\hat{x}_0$ at time $t = 0$ and compute the impulse response relative to this initial path: $\hat{x}_t \equiv x_t - x_t^{\text{init}}$ and $\hat{w}_t \equiv w_t - w_t^{\text{init}}$, where t here denotes the time since the shock occurred. Given the law of motion of productivity, the impulse response of x_t is

$$\hat{x}_t = \exp(-\alpha_x t)\, \hat{x}_0 \quad \text{for } t \geq 0$$

The paths of firm productivity before and after the shock are shown in the left panel of figure 2. In the middle panel is shown the paths of wages. To compute the black line representing the paths before the shock, I use the policy function for wages in which the realized value of shocks is 0. I use the example of a new hire whose wage is increasing over time, as explained in section 4.2. The blue lines represent the paths after the productivity shock. After the shock, wage growth accelerates and eventually overshoots before converging back to its stationary level. In the right panel is shown the pass-through of productivity shocks to wages, which is defined as the impulse response of wages normalized by the initial shock to firm productivity $\hat{x}_0$.

For a small shock, the impulse response of wages satisfies

$$\hat{w}_t \approx \int_0^t \left(rg_x(x_s, z_s)\hat{x}_s - \hat{w}_t\right) \frac{\epsilon_0}{\gamma_0} dt \tag{8}$$

where $\epsilon_0 \equiv \epsilon(V_0, x_0, z_0)$ and $\gamma_0 \equiv \gamma(w_0)$ denote the retention elasticity and the risk aversion coefficient at the time the shock occurred. This equation is an approximation of the true wage response because I abstract from changes in the ratio ϵ/γ over time.

Equation (8) can be solved in closed-form for $\hat{w}_t$ given the path for $\hat{x}_t$. Specifically, the pass-through of firm-level shocks to wages is given by

$$\frac{\hat{w}_t}{\hat{x}_0} \approx g_x(x_0, z_0)\frac{\epsilon_0}{\gamma_0} \times r\frac{\exp\left(-\alpha_x t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)}{\frac{\epsilon_0}{\gamma_0} - \alpha_x} \geq 0 \tag{9}$$

The term $g_x(x_0, z_0)$ is the change in the present value of profits induced by marginal increase in firm productivity, ϵ_0 is the retention elasticity, γ_0 is the coefficient of relative risk aversion and α_x is the degree of mean reversion of productivity. This expression can be used to study the shape of the pass-through and its size.

Consider first how equation (9) describes the shape of the pass-through, and in particular how wages are backloaded in response to a productivity shock. If wages were not backloaded, they would increase at the same pace as productivity and $\hat{w} \propto \exp\left(-\alpha_x t\right)$. Instead, wages increase at a slower pace and $\hat{w} \propto \exp\left(-\alpha_x t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)$ where the term $\exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)$ disciplines how backloaded wages are. Backloading wages after a productivity shock is optimal for firms because it induces a response of the job-to-job transition not only today but also in the future. When shocks are mean-reverting ($\alpha_x > 0$), the pass-through converges back to 0. When the shock is permanent ($\alpha_x = 0$), the pass-through converges towards $rg_x(x_0, z_0)$ and the entire shock is eventually absorbed into wages.

Consider now how equation (9) describes the size of the pass-through. In response to a productivity shock, firms face a trade-off between optimizing worker retention and providing insurance to workers. When $g_x(x_0, z_0)$ is large, the present value of profits vary significantly after productivity shocks and as a result the optimal pass-through is large. When the retention elasticity ϵ_0 is large, varying the wage of workers leads to a sharp change in the worker mobility rate. In this case, increasing the wage is an effective strategy to retain workers and the optimal pass-through is large. Remarkably, the retention elasticity is all the firm needs to know about the worker mobility decision to define its optimal policy. When the worker risk aversion γ_0 is high, indexing wages on productivity makes contracts unattractive to workers because they are risk averse. In this case the optimal pass-through is low. The optimal pass-through also depends on the persistence of shocks. To see this, note that the present value of the last term in equation (9) is $(r+\alpha_x)^{-1}$ so that the optimal pass-through falls in the degree of mean reversion of shocks α_x. To understand why less persistent shocks (higher α_x) have lower pass-through, consider the extreme example of a purely transitory shock with $\alpha_x \to \infty$. In this case, workers only generate more profits on impact and not in the future. Increasing wages has a negligible effect on the retention probability at $t = 0$ and would only help retain workers in future periods when productivity has returned to normal. The firm has no incentive to increase wages and as a result the optimal pass-through is 0.

I now characterize how productivity shocks impact the value of workers V_t, using the pass-through variable Δ_x from the promise keeping constraint.

Proposition 2. *To first-order in κ, the pass-through Δ_x of a firm productivity shock x to the value of stayers V_t satisfies*

$$(r+\alpha_x)\,\Delta_x(V,x,z) = g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)$$

where $w(V) = u^{-1}(rV)$ is the wage when $\kappa = 0$.

Proof. See appendix B.2.3. This condition is obtained from the optimality condition with respect to Δ_x when $\kappa \to 0$. □

The term $\mathcal{D}\Delta_x(V,x,z)$ in proposition 2 arises due to the dynamic nature of the pass-through. In appendix B.2.3, I derive a specific example in which $\mathcal{D}\Delta_x(V,x,z) = 0$ because sectoral productivity is constant ($\sigma_z = \rho_z = 0$) and firm productivity is normally distributed instead of log-normally. In this case, the pass-through is

$$\Delta_x(V,x,z) = (r+\alpha_x)^{-1}g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V))$$

with $g_x(x,z) = z/(r+\alpha_x)$. Appendix B.3 also shows that in a 2-period model the pass-through takes an even simpler form $\Delta_x = z\frac{\epsilon}{\gamma}u'(w)$.

To understand proposition 2, it is instructive to examine the relation between the value pass-through Δ_x and the wage pass-through $\hat{w}_t/\hat{x}_0$ from equation (9). Define the pass-through to the present value of wages as $\widehat{PV} \equiv \int_s^\infty \exp(-rt)\hat{w}_t/\hat{x}_0 dt$. We can compute it from equation (9) and get $\widehat{PV} \approx (r+\alpha_x)^{-1} g_x(x_0,z_0)\epsilon_0/\gamma_0$, which implies that $\Delta_x \approx \widehat{PV}u'(w)$. This equation shows that the change in worker value after a firm-level productivity shock is induced by a change in the present value of wages that the worker will receive at the current firm.

Relation to the Chetty-Baily formula for optimal unemployment insurance In appendix B.4, I show that the pass-through formula is reminiscent of the Chetty-Baily formula for optimal unemployment insurance (Baily, 1978, Chetty, 2006). In this literature, the planner faces a trade-off between insuring workers against unemployment risk and inducing them to search for a job. The optimal unemployment insurance takes the form of a ratio between the elasticity of the job finding rate with respect to unemployment benefits, and the coefficient of relative risk aversion of workers. In both this problem and mine, the principal tries to influence the probability that a worker finds a job and it is therefore not too surprising that the optimal policies are similar.

4.4 The differential pass-through of sectoral shocks

I now turn to the pass-through of sectoral productivity shocks and show that it differs from that of firm-level shocks because of changes in the intensity of the competition for workers. I first derive the pass-through to wages, and then to the worker value.

The pass-through of sectoral productivity shocks to wages is derived using steps similar to section 4.3 and detailed derivations are in appendix B.2.3. We get

$$\frac{\hat{w}_t}{\hat{z}_0} \approx \left(\underbrace{g_z(x_0, z_0)\frac{\epsilon_0}{\gamma_0}}_{\text{Pass-through of firm shocks}} + \underbrace{\Pi(V_0, x_0, z_0)\frac{\epsilon_{z0}}{\gamma_0}}_{\text{Differential pass-through}} \right) \times r \frac{\exp\left(-\alpha_z t\right) - \exp\left(-\frac{\epsilon_0}{\gamma_0} t\right)}{\frac{\epsilon_0}{\gamma_0} - \alpha_z} \tag{10}$$

where $\epsilon_{z0} \equiv \partial\epsilon(V_0, x_0, z_0)/\partial z$ is the cyclicality of the retention elasticity evaluated at $t = 0$. The key difference between sectoral and firm-level productivity shocks is that sectoral productivity z enters directly as an argument of the job finding rate $\lambda(v, z)$.

Equation (10) shows that the pass-through of sectoral productivity shocks to wages exceeds that of firm-level shocks if the shock persistence is the same, $\alpha_x = \alpha_z$, and two conditions are met: the firm value is positive $\Pi(V_0, x_0, z_0) > 0$ and the retention elasticity is pro-cyclical $\epsilon_z(V_0, x_0, z_0) > 0$. I now explain the intuitions that these terms capture.

After a positive sectoral productivity shocks, all firms become more profitable and therefore become eager to hire new workers. They post more vacancies and workers become more likely to switch jobs. All firms must therefore increase the wage of their workers in order to retain them. However, firms will only increase wages if workers are

Figure 3: Impulse responses from firm and sectoral productivity shocks

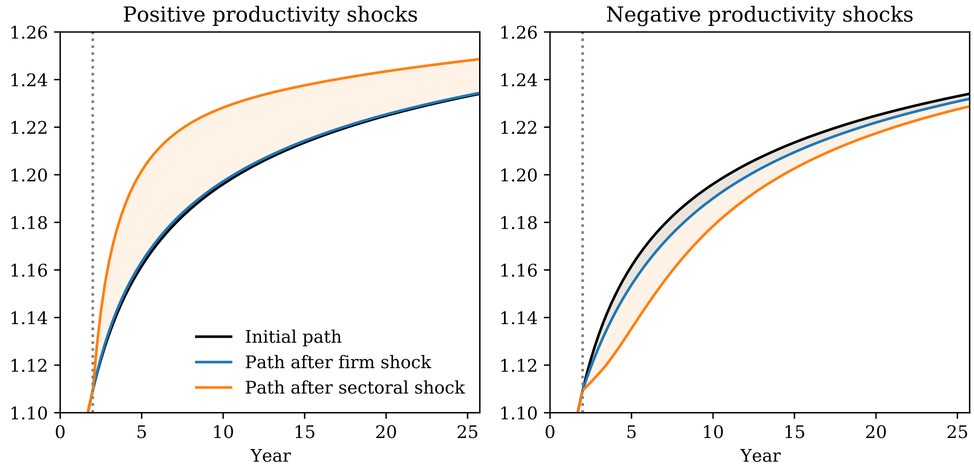
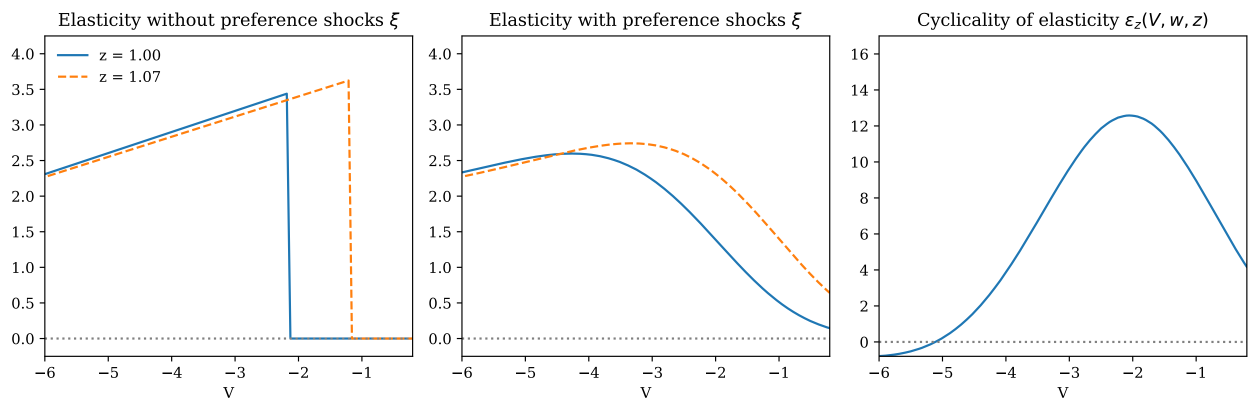


Figure 4: Level and cyclicality of the retention elasticity



worth fighting for, that is if the firm value is positive $\Pi(V_s, x_s, z_s) > 0$. If the present value of profits was zero, firms have no incentive to retain workers.

After negative productivity shocks, firms reduce wages because they become less eager to retain workers. To understand why the pass-through is amplified after negative sectoral shocks when the firm value is positive, remember that in this case wages were growing before the shock from proposition 1. After the shock, workers thus do not experience wages cuts but lower wage growth. After negative sectoral shocks, the cyclical competition for workers cools down and firms can reduce wage growth even more. Figure 3 shows the effect of positive and negative firm-level and sectoral productivity shocks on wages when the firm value is positive $\Pi(V_0, x_0, z_0) > 0$. In this figure I set $\alpha_x = \alpha_z$ and let the shocks be of the same size $\hat{x}_0 = \hat{z}_0 = 10\%$. It is possible to see that when the firm value is positive, it is the growth rate of wages that adjusts after shocks.

I now turn to the second condition that must be satisfied for the pass-through to be amplified after sectoral shocks: the retention elasticity must be pro-cyclical $\epsilon_z(V_0, x_0, z_0) > 0$. The cyclicality of the retention elasticity and the cyclicality of job-to-job transitions are closely related but do not always move in the same direction. Consider for example a worker with a relatively low wage. This worker receives more job offers in boom than usual and therefore has a pro-cyclical job-to-job transition rate. However, in a boom this worker can become so likely to leave that increasing the wage marginally does not impact her job-to-job transition rate as much as before and therefore her retention elasticity is counter-cyclical. Consider now a worker with a relatively high wage. This worker had almost no chance of changing jobs in normal time because she was too expensive to get poached. In boom, she suddenly receives more outside offers and both her job-to-job transition rate and her retention elasticity increase sharply. This worker has a pro-cyclical job-to-job transition rate and a pro-cyclical elasticity. The difference between low-wage workers and high-wage workers is illustrated in figure 4, which shows the retention elasticity without preference shocks in the left panel, the retention elasticity with them in the middle panel and the cyclicality in the right panel. One can see that the retention elasticity is more pro-cyclical for high-wage workers relative to low-wage workers. This heterogeneity will turn out to be important when I evaluate the effects of sectoral productivity shocks on workers in section 5.1.

I now derive the pass-through of sectoral productivity shocks to the worker value Δ_z.

Proposition 3. *To first-order in κ, the pass-through Δ_z of a sectoral productivity shock z to the value of stayers V_t satisfies*

$$(r + \alpha_z)\,\Delta_z(V,x,z) = \Big[\underbrace{g_z(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))} + \Pi(V,x,z)\frac{\epsilon_z(V,x,z)}{\gamma(w(V))}}_{\text{Pass-through to present value of wages}}\Big]u'(w(V))$$

$$+ \underbrace{\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z) - V)}_{\text{Change in expected gains from J2J transitions}} + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z) \equiv \frac{\partial \epsilon(V,x,z)}{\partial z}$ *and* $\lambda_{wz}(v(V,z),z) \equiv \frac{\partial \lambda_w(v,z)}{\partial z}\big|_{v=v(V,z)}$.

Proof. See appendix B.2.3. □

Proposition 3 shows how changes in the intensity of the competition for workers impact workers after sectoral shocks. First, the worker value changes due to the larger response of wages. Second, sectoral productivity shocks impact the worker value directly through changes in the probability of finding a job. In booms, the competition for workers heats up and workers are more likely to find a new job so that $\lambda_{wz}(v(V,z),z) > 0$. When they switch, they gain a value $v(V,z) - V$. Workers therefore benefit in booms because it increases the probability of a job-to-job transition. In downturns, the opposite is true.

5 Quantitative analysis

The previous section established that the retention elasticity is a critical determinant of the pass-through of productivity shocks to wages. In this section, I calibrate the quantitative model using administrative data from France with a focus on moments that are informative about this elasticity. I then use it to evaluate how much insurance firms provide to workers over the cycle.

5.1 Quantification

I quantify the model by matching moments of the matched employer-employee data from France between 2008 and 2019.

Quantification strategy I quantify the model in two steps: first, I set some parameters externally; second, I infer the remaining model parameters by moment matching.

The model parameters set externally are the utility function, the discount factor, the matching function and the degree of firm commitment Φ. The utility function is CRRA with coefficient $\gamma = 1.5$, following Balke and Lamadon (2022) and the discount factor is $\beta = 0.99$ to match an annual interest rate of 4%. The matching function is Cobb-Douglas

$$\mathcal{M}(\phi_u + \kappa\phi_e, \phi_v) = B\left(\phi_e + \kappa\phi_u\right)^{\nu}\phi_v^{1-\nu}$$

with $\nu = 0.5$, which is an intermediate estimate between Menzio and Shi (2011) and Shimer (2005). $B = 0.26$ is calibrated to get a market tightness $\phi_v/(\phi_e + \kappa\phi_u)$ of 0.6, following Hagedorn and Manovskii (2008), given the job finding rate in my model[6].

The degree of firm commitment Φ is set using estimates of firing costs. It is standard in the literature to justify firm commitment on the ground that firms have reputation concerns. This justification however only holds for relatively large and well-known firms. Instead, I argue that firing costs are a better proxy for firm commitment power. In my model firms only want to walk away from the contract if their value falls below the cost of firing the worker. Layoffs are tightly regulated in France and can lead to lawsuits and

[6] I do not have vacancy data for France and thus cannot estimate the matching function directly.

large compensations for workers. The parameter Φ captures the expected cost of a layoff, including severance payments and penalties that firms pay when layoffs are challenged in court. I calibrate the firing costs to 4 months of labor earnings, $\Phi = 1.75$, following a methodology from Bentolila and Bertola (1990) that I update with recent data from the International Labor Organization (see appendix A.5 for details).

The other model parameters are inferred by matching moments in the French data and in model-simulated data. Specifically, I simulate a panel of workers across different industries in the model and estimate the exact same set of moments in the model and in the data. The estimated parameters are the degree of mean reversion of productivity shocks α_x, α_z, their volatility σ_x, σ_z, the dispersion of fixed productivity across firms $\sigma_{\overline{x}}$, the vacancy posting cost k, the value of home production b, the exogenous separation rate δ, the search efficiency on the job κ, and the volatility of preference shocks σ_{ξ}. My estimates of shocks persistence in the data are too noisy at the sector level so in the estimation I set $\alpha \equiv \alpha_x = \alpha_z$ and use the persistence of productivity at the firm level to estimate α. This leaves 9 parameters that will be estimated using 9 moments in the data.

Table 2 shows the moments used in the estimation. I use moments on average labor market flows, on the tenure profile of wages and job-to-job transitions and on productivity. I find that workers spend on average 14 months non-employed, which implies a low quarterly job finding rate of 20%. Separations into employment and non-employment are both much lower than in the United States. Remarkably, only 51% of workers changing jobs experience a positive wage change when they do so, highlighting the critical importance of preference shocks for France. Appendix A contains more details on how these moments are computed.

Parameter values The value of estimated parameters are shown in table 3. I will describe the results of the inference exercise with an informal discussion of what moments influences which parameters most.

The value of home production b is influenced by the tenure profile for wages. Wages tend to increase until profits are null, and therefore until they are approximately equal to the average value of output. Since we know where wages converge to, the tenure profile of wages gives us the starting wage of new workers. This starting wage in the model is tightly connected to the value of home production b because the latter determines the market in which workers from unemployment search for a job. The relatively low tenure profile for wages in the data (10% cumulative change after 25 years of tenure from figure 1) implies a high value of home production $b = 0.95$, or 77% of the average wage (0.95/1.23). Interestingly, this estimates lies in the range of values (47% to 96%) proposed

Moments	Data	Model
Average duration non-employed in months	14.3 (0.063)	14.6
Annual separation rate into non-employment	5.5% (0.071%)	5.2%
Annual job-to-job transition rate	6.6% (0.10%)	6.6%
Share of job-to-job transitions with a positive wage change	51% (0.39%)	51%
Tenure profile of wages at 7.5 years	7.1% (0.04%)	7.0%
Tenure profile of job-to-job transitions at 7.5 years	- 6.6% (0.04%)	- 6.4%
s.d. of firm productivity growth	0.30 (0.026)	0.30
s.d. of sector productivity growth	0.057 (.019)	0.057
Annual persistence of firm productivity	0.81 (0.01)	0.81

Table 2: Targeted moments in data vs. model

by Chodorow-Reich and Karabarbounis (2016) for the United States, even though France has much higher unemployment benefits.

The vacancy posting cost k influences the job finding rate, and is thus pinned down by the duration of non-employment. The vacancy posting cost $k = 0.4$, or 1 month of average wages ($3 \times 0.4/1.23$), is slightly larger than estimates for the United States. For instance, Hagedorn and Manovskii (2008) report an estimate of 2.5 weeks of wages.

The average separation rate into non-employment pins down the exogenous separation probability δ. In the model, matches separate endogenously when productivity falls sufficiently low relative to the value of home production b, and exogenously when the separation shock with probability δ occurs. The relatively high value of b means that matches are quite likely to endogenously separate and as a result a significant fraction of separations are endogenous (about 40%). This means that a significant share of separations observed in the data can be attributed to negative shocks occurring at the firm or sector level, as opposed to shocks occurring at the worker level.

The average job-to-job transition rate pins down the search efficiency of employed workers κ. My estimate of 0.65 is consistent with the estimates of 0.53 from Balke and Lamadon (2022) for Sweden.

The share of workers experiencing positive wage growth when they switch job pins down the volatility of preference shocks σ_ξ. In the model, workers move to lower paying jobs when they switch to firms with higher productivity because these firms have a steeper tenure profile for wages, as in Burdett and Coles (2010). However, this mecha-

Parameters	Value	Parameters	Value
Value of home production b	0.95	Vacancy posting cost k	0.4
Search efficiency on the job κ	0.65	Exogenous separation rate δ	0.0045
Volatility of preference shocks σ_ξ	0.22	Dispersion in fixed productivity $\sigma_{\bar{x}}$	0.11
Volatility of firm productivity σ_x	0.14	Volatility of sectoral productivity σ_z	0.035
Persistence of firm and sector productivity $1-\alpha$	0.93		

Table 3: Estimated model parameters

nism alone does not generate enough transitions with negative wage changes compared to the data. Instead, some workers in the model will switch to lower paying jobs because they receive high preference shocks for switching jobs.

The tenure profile for job-to-job transitions influences most directly the dispersion in firm fixed productivity $\sigma_{\bar{x}}$. When there is a lot of dispersion (high $\sigma_{\bar{x}}$), firms with low fixed productivity are very exposed to the competition for workers because they pay their workers very low wages. These are the firms for which the tenure profile for job-to-job transitions will be the steepest. Firms with higher than average productivity are almost isolated from this competition for workers because they pay high wages and thus exhibit a flat profile irrespective of the value of $\bar{x}$. As a result, a steeper profile of job-to-job transitions in the data is indicative of more dispersion in firm productivity.

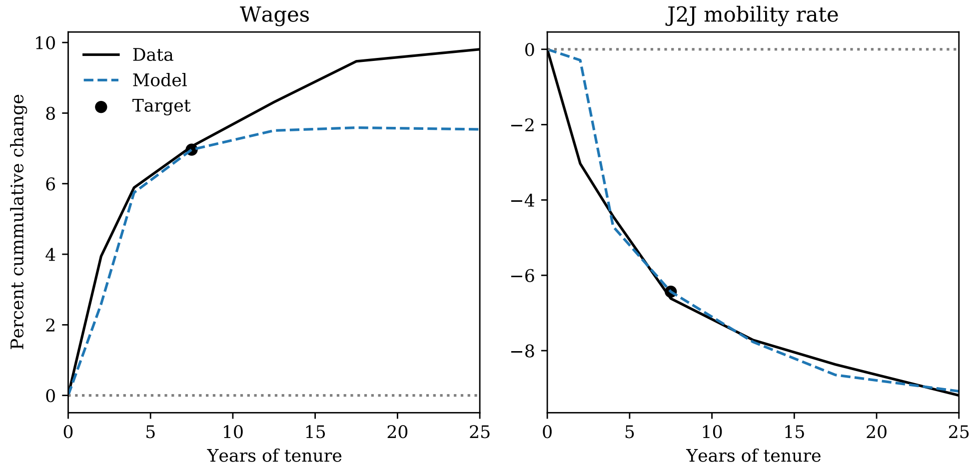
Finally, the volatility and persistence of productivity $\sigma_x, \sigma_z, \alpha$ are estimated directly from data on productivity. The volatility of firm productivity is 4 times larger than the volatility of sector productivity. Productivity shocks are moderately persistent, with a quarterly AR-1 coefficient of 0.93, which is equivalent to about 0.75 annually.

Untargeted moments As validation of the model, I assess whether it matches the differential response of wages and job-to-job transitions to firm-level and sectoral productivity shocks documented in section 2. I also compare the tenure profile for wages and job-to-job transitions that were partially targeted in the estimation.

Figure 5 shows the tenure profiles in the data (solid black lines) and in the model (dotted blue lines). The solid dots represent the moments targeted in the estimation. The model accounts very well for the entire tenure profile of job-to-job transitions, and undershoots slightly the tenure profile for wages at long horizons.

Table 4 shows estimates of the response of wages and job-to-job transitions to firm-level and sectoral shocks. The first column repeats the data estimates from table 1 while

Figure 5: Tenure profiles in data vs. model



Moments	Data	Model
Response of wages to firm-level productivity shock $\theta^{w,x}$	4.7%	5.3%
Response of wages to sectoral productivity shock $\theta^{w,z}$	18.5%	15.9%
Response of job-to-job transitions to firm-level productivity shock $\theta^{J2J,x}$	- 1.7 pp	- 6.2 pp
Response of job-to-job transitions to sectoral productivity shock $\theta^{J2J,z}$	4.0 pp	8.0 pp

Table 4: Response to shocks in data vs. model

the second column reports the same estimates from the model. The model accounts for 76% of the larger pass-through of sectoral shocks to wages ($(15.9 - 5.3)/(18.5 - 4.7)$). The model also accounts for the difference in sign of the response of job-to-job transitions to firm and sectoral shocks, but it overstates the magnitude relative to the data.

Why does the model get the differential response of wages and job-to-job transitions right? I now briefly build on the intuitions from section 4 to explain this result.

In response to firm-level shocks, firms could keep wages constant and provide maximum insurance to workers. However, this strategy is not optimal because with such a pass-through of 0% the worker is leaving at a constant rate. Instead, the firm can increase profits by paying the worker relatively more when productivity is high, and relatively less when productivity is low. This strategy of positive pass-through induces workers to stay precisely when they generate the most profits. For this reason, in response to positive firm-level shocks to productivity, wages increase and job-to-job transitions fall.

In response to sectoral shocks, the incentives of firms to pass-through shocks to wages are shifted because sectoral shocks also influence the intensity of the cyclical competition for workers. Specifically, after a positive shock to sectoral productivity, all firms are more profitable and want to attract new workers. They post more vacancies and workers are more likely to find a new job. As a result, existing firms face further incentives to increase their workers' wages: not only their workers are more productive and generate more profits, but they are also more likely to leave. Firms thus increase the wage of their existing workers more aggressively to retain them. The response of job-to-job transitions to sectoral shocks turns out to be ambiguous. On the one hand, workers receive more outside offers from new firms. On the other hand, firms are trying harder to retain them. It turns out that in the quantitative model, as in the data, the first force dominates and job-to-job transitions are positively correlated with sectoral shocks.

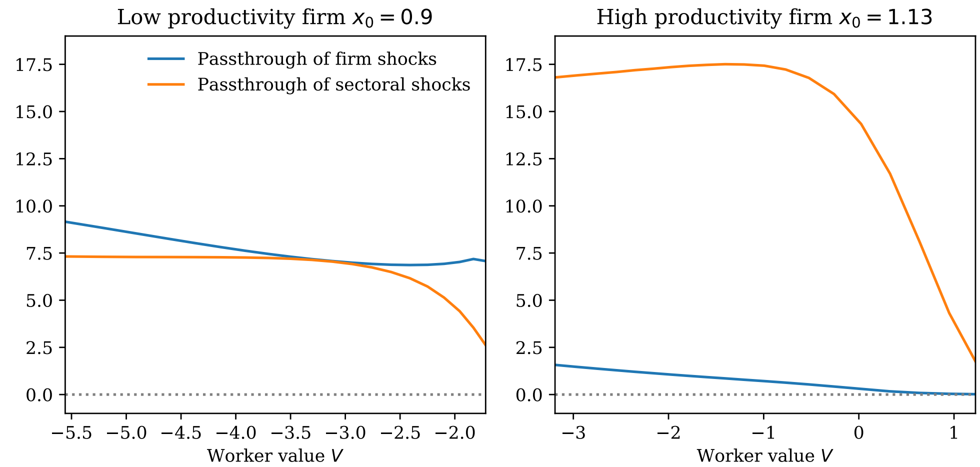
Heterogeneous pass-through The average pass-through estimates reported in table 4 hide significant differences across workers with different states $(V, \overline{x}, x, z)$. For instance, figure 6 shows the pass-through of firm and sectoral shocks for a match with relatively low fixed productivity $\overline{x} = 0.9$ and for a match with relatively high fixed productivity $\overline{x} = 1.13$. Workers in low-productivity matches (low $\overline{x}$) experience a pass-through of firm and sectoral shocks that is almost identical, whereas workers in high-productivity matches (high $\overline{x}$) experience a lower pass-through of firm-level shocks and a much higher pass-through of sectoral shocks. To understand why there is no amplification of the pass-through for workers in low productivity matches ($\overline{x} = 0.9$), it is useful to remember from section 4.4 that the pass-through of sectoral shocks is amplified when firms have positive values and the retention elasticity is pro-cyclical. Workers in low-productivity matches generate little profits for firms, and their retention elasticity is only mildly pro-cyclical because their wages are low. By contrast, workers in high productivity matches ($\overline{x} = 1.13$) generate a lot of profits and have high wages. As a result, they are unlikely to get poached in bad time, but quite likely in good time and their elasticity is strongly pro-cyclical. Intuitively, after a positive sectoral productivity shock high-productivity firms realize that their workers are now getting poached and that they must increase their wage to retain them.

5.2 Firm commitment and firing costs

I use the quantitative model to measure how much commitment power firms effectively have and how much insurance workers receive as a result of this.

As in Thomas and Worrall (1988), the limited commitment of firms implies that after large negative shocks wages sometimes have to fall sharply to prevent firms from walking

Figure 6: Pass-through of firm and sectoral productivity shocks for different $\overline{x}$



away from the deal. If wages did not fall, then firms would be better off firing the worker and walking away from the contract. Specifically, remember that optimal contracts imply that after any history it must be that

$$\Pi(V, s_t) \geq -\Phi$$

where Φ captures the degree of commitment of firms, and is calibrated using estimates of firing costs. This constraint is more likely to bind when the worker value V is high because this implies that wages must be high, and when firm and sectoral productivity x, z are low because this implies that revenues are small.

Proposition 1 showed that firms increase wages until the firm value $\Pi(V, s_t)$ or the retention elasticity $\epsilon(V, s_t)$ reaches 0. This implies that over the duration of a match profits fall towards 0 and as a result this constraint becomes more likely to bind. Intuitively, firms compete for workers by increasing their wages over time and eventually workers extract all the surplus from a match. But then if productivity falls, the firm does not have any buffer to absorb the shock and the firm participation constraint binds.

I first assess whether in my quantitative model calibrated for France the limited commitment constraint of firms is likely to be binding. This depends on the size of firing costs, the volatility of productivity shocks and any parameter governing how fasts profits fall towards 0. I report in table 5 the fraction of firms hitting this constraint each year. I find that only 0.6% of firms hit this constraint, which means that firing costs are sufficiently large in France that firms effectively have almost full commitment.

	Firms at constraint	Pass-through firm	Pass-through sector	Sep. rate
Baseline model ($\Phi = 1.75$)	0.6%	5.3%	15.9%	5.2%
Low firing costs ($\Phi = 0.175$)	14.7%	9.0%	22.9%	8.3%

Table 5: Counterfactual with lower firing costs

Significantly less insurance with lower firm commitment Φ I now reduce the degree of firm commitment Φ using estimates of firing costs for the United States. Effectively, I ask what would happen if a country like France with high firing costs were to implement labor market liberalization policies lowering firing costs to a level similar to the U.S. Appendix A.5 documents that the United States have much lower firing costs and that a reasonable estimates for them is 2 weeks of wage, equivalent in the model to $\Phi = 0.175$.

When firing costs are reduced, firms are a lot more likely to hit their participation constraint. Specifically, this constraint is binding for 14.7% of firms each quarter. As a result, the pass-through of firm-level shocks and sectoral shocks rise significantly, from 5.3% to 9.0% and from 15.9% to 22.9% respectively. Interestingly, the pass-through of firm-level shocks becomes counter-cyclical when firing costs are lowered: it is higher in downturns when sectoral productivity is below average, than in booms when sectoral productivity is above average. Specifically with high firing costs ($\Phi = 1.75$), the pass-through is roughly a-cyclical in the model (5.1% in downturns vs. 5.4% in booms), which is consistent with my estimates from the data reported in appendix A.3. With low firing costs ($\Phi = 0.175$), the pass-through is significantly larger in downturns than in booms (9.8% vs. 8.2%). This pattern occurs because in downturns sectoral productivity is low relative to wages. As a result, profit margins are small and firms are more likely to hit the firm participation constraint after negative firm-level shocks.

Finally, lowering firing costs, and therefore firm commitment power, leads to an increase in the separation rate into non-employment. The separation rate rises significantly from 5.2% to 8.3%, because the surplus from matches is smaller when firing costs are low and therefore matches are more likely to be efficiently terminated. The surplus from a match depends on the ability of firms to provide insurance to workers. If firms cannot insure workers against productivity shocks, the surplus from matches is low and matches with low productivity are more likely to be terminated. If firms can insure workers, matches are less likely to be terminated even if productivity is low because keeping the match alive allows firms to smooth the worker consumption, whereas terminating the match would induce a sharp fall in the worker consumption.

These results point to a novel role of firing costs when wage contracts are endogenous:

they enhance firm commitment power and improve insurance for workers.

5.3 Wage inequality over the cycle

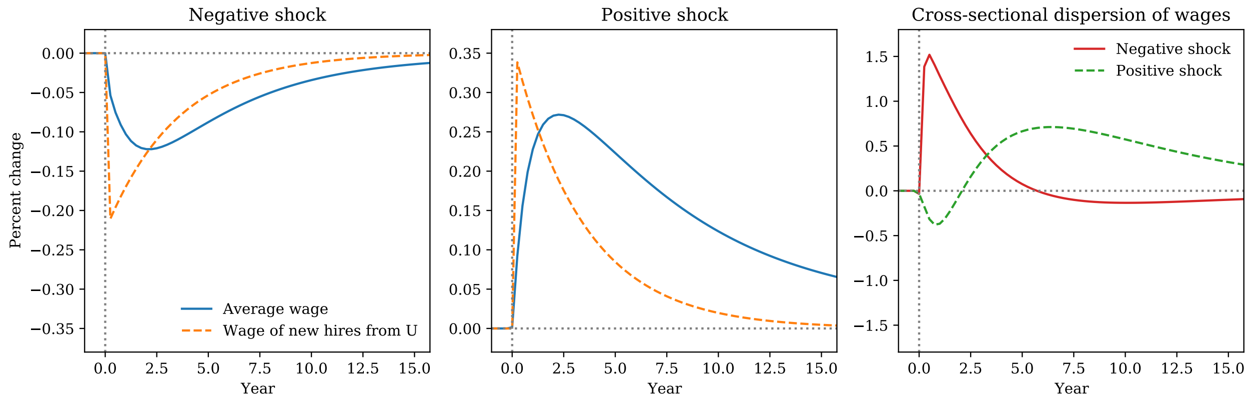
Sectoral fluctuations in productivity are not only a source of risk for employed workers, but also for workers transitioning through unemployment. In a downturn, these workers start with a much lower wage compared to workers who kept their job and experience a much more progressive fall in wages due to the insurance provided by contracts.

Figure 7 illustrates this differential response of wages for newly hired workers out of unemployment and for continuously employed workers. The left panel shows the percent change of wages for new workers out of unemployment and the average wage after a 1% negative and mean-reverting shock to sectoral productivity. The wage of workers out of unemployment falls on impact by about 0.2%, and then gradually recovers following the path of productivity. The wage of continuously employed on the other hand only falls gradually, which is reflected in the trajectory of the average wage. This differential dynamic of wages leads to an increase in wage inequality during downturns, as illustrated by the right panel. The cross-sectional standard deviation of log wages among employed workers rises by about 1.5% in downturn because a large mass of new hires start with relatively low wages, while the wage of continuously employed worker adjusts slowly. After 6 years, wage inequality has reverted back to its pre-shock level and overshoots slightly as the wage of new hires from unemployment has recovered faster than the average wage.

The middle panel shows the response of wages to a 1% mean-reverting positive sectoral productivity shocks. The dynamics of wages are symmetric but larger than in the left panel because the pass-through is larger for positive than for negative shocks. The dynamics of wage inequality in the right panel is symmetric between negative and positive shocks but the short-term response is much smaller for positive than negative shocks, and the long-term response much stronger. The reason for this difference is that wage gains are a lot more persistent than wage cuts (after a temporary increase in sectoral productivity, workers move to a region of the state space where the retention elasticity $\epsilon(V, s)$ is close to 0 so wages remain stable from proposition 1).

In order to quantify the cyclicality of wage inequality, I compute the cross-sectional standard deviation of log wages on average, in downturns (lower than average sectoral productivity) and in booms (higher than average). Wage inequality is 12% larger in downturns than in booms (0.064/0.057). This result is qualitatively consistent with evidence from Coibion, Gorodnichenko, Kueng and Silvia (2017) who show that wage inequality

Figure 7: Impulse response of wage inequality to sectoral productivity shocks



Note: impulse response to a 1% shock to sectoral productivity.

	Average	Downturns	Booms
Cross-sectional sd. of log wages	0.060	0.064	0.057

Table 6: Wage inequality over the cycle

rises in response to contractionary monetary policy shocks.

6 Extension: introducing risk-free bonds

An important assumption I made so far in this paper is that workers do not have access to financial markets and consume their wage. In this section I show that when workers have access to risk-free bonds, the wage contracts offered by firms changes significantly: they lead to much more wage backloading and feature a new trade-off between worker retention and precautionary savings. When trades in risk-free bonds are private information, workers borrow more than what firms would like so firms increase the pass-through of productivity shocks to wages in order to make workers save for precautionary reasons.

6.1 Environment

I study a 2-period version of the model with risk-free bonds. A 2-period model without risk-free bonds is described in appendix B.3.

I assume that workers do not have preference shocks and do not quit voluntarily into unemployment. Matches can still separate with exogenous probability δ.

The timing works as follows: at $t = 0$, a unit mass of workers is matched with firms.

These workers have promised value V_0. Firm-level and sectoral productivity are normalized to 1 at $t = 0$. At the beginning of $t = 1$, firm-level and sectoral productivity shocks x and z are realized and some matches separate exogenously with probability δ. After this, firms post vacancies to poach workers from existing matches, employed workers search for new jobs and new matches are formed. Firms then produce and workers receive their wage if they are employed, and home production b if they are unemployed. Workers can trade risk-free bonds when they receive their wage at $t = 0$ and these bonds are due at $t = 1$ when wages are paid. Workers consume at the end of each period.

Note that the timing is slightly different than in the quantitative model because I assume that workers can switch jobs before production occurs and wages are paid.

Workers can trade risk-free bonds a at rate $R = 1/\beta$. They are not able to default on this asset. I assume first that firms observe and control this choice, and will relax this assumption in section 6.3. Firms can pay severance payments τ to workers in the event of a separation into unemployment. I assume that if a firm commits to a level of severance payment but does not fulfill its promise ex-post, it incurs the cost Φ.

In the 2-period model with risk-free bonds, it is convenient to index labor markets by the wage that workers receive from poachers $\tilde{w}_1$ instead of the value that workers receive. Labor markets are also indexed by the asset a of workers who search there. The expected profit from posting a vacancy in market $(\tilde{w}_1, a)$ at $t = 1$ is

$$\Pi_0(\tilde{w}_1, a, z) = -k + \lambda_f(\tilde{w}_1, a, z)\left[x^e z - \tilde{w}_1\right]$$

where x^e is the firm-level productivity of a new entrant. The free entry condition $\Pi(\tilde{w}_1, a, z) = 0$ implies that the vacancy filling rate $\lambda_f(\tilde{w}_1, a, z) = \lambda_f(\tilde{w}_1, z)$ is independent of assets, and from the matching function so is the job finding rate $\lambda_w(\tilde{w}_1, z)$.

A contract specifies wages $\{w_0, w_1(x, z)\}$ at $t = 0$ and at $t = 1$ for each realization of firm-level and sectoral productivity, as well as severance payments τ paid after an exogenous separation into unemployment. Given the contract and given a level of assets a, the value of the worker satisfies

$$\begin{aligned} V = {} & u(w_0 - a) + \delta\beta u(b + aR + \tau) \\ & + (1-\delta)\beta\mathbb{E}_{x,z}\left[\max_{\tilde{w}_1}(1 - \kappa\lambda_w(\tilde{w}_1, z))u(w_1(x, z) + Ra) + \kappa\lambda_w(\tilde{w}_1, z)u(\tilde{w}_1 + Ra)\right] \end{aligned} \tag{11}$$

The worker search policy $\tilde{w}_1(w_1(x, z), a, z_2)$ satisfies the optimality condition

$$\lambda_w(\tilde{w}_1, z_2)\left[u(\tilde{w}_1 + Ra) - u(w_1(x, z) + Ra)\right] + \lambda_w(\tilde{w}_1, z_2)u'(\tilde{w}_1 + Ra) = 0 \tag{12}$$

The optimal contract maximizes the present value of profits subject to a promise keeping constraint, the incentive compatibility constraint for search and the firm participation constraints. It solves

$$\max_{w_0, w_1(x,z), \tau, a} \quad 1 - w_0 + \beta(1-\delta)\mathbb{E}_{x,z}\left[(1 - \kappa\lambda_w(\tilde{w}_1, z))\,(xz - w_1(x,z))\right] - \beta\delta\tau \tag{13}$$

subject to

$$\begin{aligned}
&\text{(PK)} && V_0 = u(w_0 - a) + \delta\beta u(b + Ra + \tau) \\
& && \quad +(1-\delta)\beta\mathbb{E}_{x,z}\left[(1 - \kappa\lambda_w(\tilde{w}_1, z))u(w_1(x,z) + Ra) + \kappa\lambda_w(\tilde{w}_1, z)u(\tilde{w}_1 + Ra)\right] \\
&\text{(IC-v)} && \tilde{w}_1 = \tilde{w}_1(w_1(x,z), a, z) \\
&\text{(PC-F)} && xz - w_1(x,z) \geq -\Phi \\
&\text{(PC-F2)} && -\tau \geq -\Phi
\end{aligned}$$

I characterize the optimal contract in section 6.2 and discuss the implications of hidden trade in section 6.3.

6.2 Firms use debt to backload wages even more

I first show that introducing risk-free bonds enables firms to backload wages even more. When firms have limited commitment however, the extent to which firms can backload wages depends a precautionary savings motives.

To build some intuition for the results in this section, it is useful to remember the trade-off that firms face when workers had no access to financial markets. Firms backload wages to prevent workers from leaving for another job, because it makes them apply for jobs with a lower job finding rate. However, when workers have no access to financial markets backloading wages implies backloading consumption too, and such contracts are relatively unattractive to workers with concave preferences. Firms would have to pay workers a higher average wage when the wage profile is backloaded if it wants to hire them. As a result, wages are only partially backloaded as shown by proposition 1.

Introducing trades in risk-free bonds expands the set of feasible contracts available to firms. In particular, it is now possible for firms to smooth consumption even when wages are backloaded by making the worker borrow. Because of this, firms choose to backload wages more and the job-to-job mobility rate falls drastically. As I will show in this section, whether wages are entirely backloaded or not now depends on the degree of firm commitment power and on a new precautionary savings motive.

It might be surprising that introducing observable trades in risk-free bonds changes the optimal allocation[7]. The reason is that it relaxes a critical assumption implicitly made in the model with hand-to-mouth workers. Specifically, when there is no risk-free bond firms cannot track workers when they change jobs, and as a result it is not feasible for workers to make a payment to a previous employer. When workers can trade risk-free bonds on which they cannot default, tracking workers across jobs becomes feasible and firms thus face fewer constraints when they design wage contracts.

In order to make these points as transparent as possible, I assume in this section that utility is CARA $u(c) = -\exp(-\gamma c)/\gamma$. This assumption implies from condition (12) that the search policy $\tilde{w}_1(w_1(x,z),z)$ is independent of assets a. I will relax this assumption in section 6.3 when I discuss the implications of hidden trades.

Combining the optimality conditions of the contracting problem gives

$$\frac{u'(b+Ra+\tau)}{u'(w_0-a)} = 1+\mu_U$$

$$\frac{u'(w_1(x,z)+Ra)}{u'(w_0-a)} = 1+\frac{\kappa\partial_{w_1(x,z)}[\lambda_w(\tilde{w}_1(w_1(x,z),z),z)]\,(xz-w_1(x,z))+\mu_{xz}}{1-\kappa\lambda_w(\tilde{w}_1(w_1(x,z),z),z)}$$

$$\begin{aligned} u'(w_0-a) &= \delta u'(b+Ra+\tau)+(1-\delta)\mathbb{E}_{x,z}\left[u'(w_1(x,z)+Ra)\right] \\ &+(1-\delta)\mathbb{E}_{x,z}\left[\kappa\lambda_w(\tilde{w}_1(w_1(x,z),z),z)\left(u'(\tilde{w}_1(w_1(x,z),z)+Ra)-u'(w_1(x,z)+Ra)\right)\right] \end{aligned} \tag{14}$$

where μ_{xz} and $\delta\beta\mu_U$ are the Lagrange multipliers of the firm participation constraints. The first equation is the optimality condition for severance payments τ. The second equation is the optimal wage growth condition. The third condition is the optimal saving condition, which here turns out to be the worker's Euler equation.

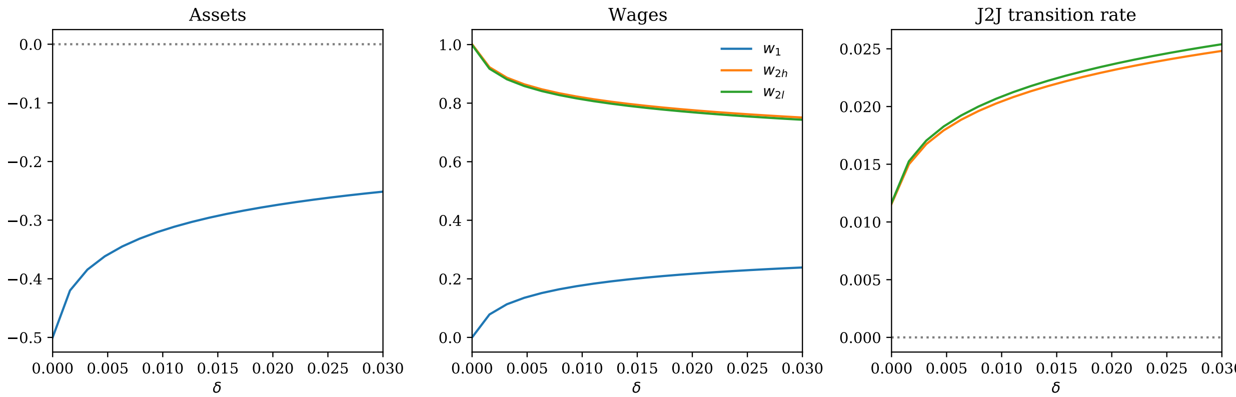
Full backloading with firm commitment Consider first the case with full commitment ($\Phi \to \infty$). The two participation constraints of the firm drop out so $\mu_{xz} = \mu_U = 0$.

The optimality condition for severance payments shows that the firm insure the worker against unemployment risk since $c_0 \equiv w_0 - a = b + Ra + \tau = c_U$. Combining the three optimality conditions, we find that

$$w(x,z) \geq xz$$

[7] For example, in the optimal unemployment insurance literature (Hopenhayn and Nicolini, 1997) trades in risk-free bonds are irrelevant if they are observable to firms. The reason is that in this literature the optimal contract pins down the allocation uniquely, but this allocation can be implemented using various transfer and savings schemes. It is then without loss of generality to focus on the scheme where the agent does not borrow or save, and instead consumes the transfer.

Figure 8: Optimal contract with risk-free bonds: comparative statics with respect to δ



Note: illustrative example with two productivity states x_h and x_l.

so that workers get at least the value of output tomorrow. This is an extreme form of backloading in which firms pay more than the value of output to workers tomorrow irrespective of their promised value V_0 or wage at $t = 0$. The wage at $t = 0$ is pinned down by the promise keeping constraint and depends on V_0. It can even be negative, meaning that workers have to pay an upfront fee to firms. Because wages are so backloaded, the job-to-job transition rate falls drastically.

Consumption satisfies

$$c(x,z) \leq c_0 = c_U \leq \tilde{c}(x,z)$$

where $\tilde{c}(x,z)$ is the consumption of a worker in state (x,z) after a job-to-job transition. This shows that consumption is frontloaded if the worker does not switch job, and backloaded if the worker switches job.

This result is closely related to Stevens (2004) who studies optimal wage contracts with complete financial markets. Remarkably, risk-free bonds are almost sufficient here to achieve the same results even though there are many sources of risk (e.g. unemployment, productivity shocks). This is because firms can almost replicate the complete market allocation using risk-free bonds and severance payments.

Partial backloading with limited firm commitment Consider now the case with limited firm commitment ($\Phi < \infty$). In this case firms can only insure workers against negative events such as unemployment risk and negative productivity shocks if it is in their interests ex-post. In the limit case with $\Phi = 0$, firms cannot commit to make losses after adverse shocks so that $\tau = 0$ and $w(x,z) \leq xz$.

As a result, workers dislike entering period 1 with too much debt because in the event of a negative shock or unemployment shock they will have very little income to repay it and will as a result consume very little. This prevents firms from backloading wages too much because it would make the worker either borrow or backload consumption too much at $t = 0$. This precautionary savings motives prevent workers from borrowing too much in anticipation of future income because this income is uncertain.

Figure 8 illustrates how the path of consumption changes as the exogenous separation rate δ rises from 0 to 3% when Φ is set to 0. When the risk of separation into unemployment increases, the optimal contract implies less borrowing since workers face the risk of having to repay the debt while unemployed. As a result, wages become less backloaded and the job-to-job transition rate increases.

6.3 Hidden trade

I have assumed so far that firms could observe and thus control the asset choice of agents. I now assess whether the allocation changes when agents can privately access financial markets, that is they are hidden trades.

I solve the problem with hidden trade using the first-order approach following Werning (2001) and Abraham and Pavoni (2008). With hidden trade, the worker privately chooses in which labor market to apply $\tilde{w}_1$ and how much assets to hold a to maximize her present value (11). Taking the first-order condition with respect to asset a gives the Euler equation (14). The optimal contract now solves the problem (13) with the Euler equation (14) as an additional constraint.

With CARA utility the relaxed problem without hidden trade (13) solves the problem with hidden trade. To see this, note that the optimality condition with respect to assets a in the relaxed problem is the agent's Euler equation. Therefore the solution to the relaxed problem is also feasible in the problem with hidden trade, and since we can always do better in a relaxed problem it is also the solution with hidden trade. Intuitively, with CARA utility there are no wealth effects in that the level of assets does not influence the worker's search decision. As a result, given a choice for v, the worker and firm preferences towards savings are aligned.

For a general utility function there are profitable joint deviations for the worker. To understand this, it is useful to consider the optimal choice of assets of the firm in the relaxed problem (13) for a general $u(c)$ when firms control the level of assets directly. The optimality condition for assets a becomes, using the envelope theorem,

$$\begin{aligned} u'(w_0 - a) = {} & \delta u'(b + Ra + \tau) + (1-\delta)\mathbb{E}_{x,z}\left[u'(w_1(x,z) + Ra)\right] \\ & + (1-\delta)\mathbb{E}_{x,z}\left[\kappa\lambda_w(\tilde{w}_1(w_1(x,z),a,z),z)\left(u'(\tilde{w}_1(w_1(x,z),a,z) + Ra) - u'(w_1(x,z) + Ra)\right)\right] \\ & - \kappa\partial_a\lambda_w(\tilde{w}_1(w_1(x,z),a,z),z)u'(w_0 - a)(1-\delta)\beta\mathbb{E}_{x,z}\left[xz - w_1(x,z)\right] \end{aligned}$$

Compare this optimality condition with the Euler equation (14). The first two lines of these conditions are identical. The last line shows that firms take into account that assets influence the search decision of workers, and therefore their job-to-job transition rate $\partial_a \lambda_w(\tilde{w}_1(w_1(x,z),a,z),z) \neq 0$. When firms make positive expected profits $\mathbb{E}_{x,z}[xz - w_1(x,z)]$, they alter the level of assets in a way that reduces the job-to-job transition rate so as to retain workers. Workers with more assets search for jobs with a higher wage $\partial_a \tilde{w}_1(w_1(x,z),a,z) > 0$. Since the job finding rate $\lambda_w(w,z)$ is decreasing in w, firms increase the worker's savings in order to reduce their job-to-job transition rate.

Intuitively, workers enter period 1 with some assets a. The lower this level of asset, the higher the marginal utility of consumption. With low assets a, workers would rather search for jobs that are easier to get and deliver a smaller increase in consumption, than jobs that are difficult to get. Conversely, if workers enter with a high level of assets they are willing to search for jobs that they are unlikely to get but deliver a higher payoff. Said differently, searching for a new job is comparable to buying a lottery ticket, and workers are more willing to enter a risky lottery if their marginal utility is low (asset level is high) than otherwise. Because of this, when firms want to retain workers they make them borrow less at $t = 0$ and enter period 1 with a relatively low level of debt.

What would workers choose with hidden trade? In the allocation with observable trades workers are borrowing constrained because their Euler equation is not satisfied: relative to firms, they would prefer to borrow more at $t = 0$ to increase consumption. The joint deviation is therefore to borrow more at $t = 0$, enter period $t = 1$ with more debt and search in a labor market with a lower wage $\tilde{w}_1$ and a higher job finding rate. In response, firms try to induce workers to borrow less at $t = 0$ by increasing the pass-through of productivity shocks to wages so as to increase the precautionary savings motive of workers.

7 Conclusion

This paper studies the determinants of the pass-through of firm-level and sectoral productivity shocks to wages. I build a model in which firms set the optimal pass-through based on a trade-off between insuring workers against shocks and responding to a cyclical competition for workers. The quantitative model accounts well for the patterns of pass-through documented in the data. An implication of this model is that firing costs influence the wage contracts that firms offer to workers and therefore the volatility of wages over the cycle.

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Appendix

Contents

A Data appendix

A.1 Sample construction

I use administrative data provided by the CASD in France between 2008 and 2019. My analysis relies on two main files:

a) the panel version of the "DADS tous salariés" database, containing detailed information about employment history for 1/12th of the French population every year;

b) "FARE" database, with annual information about firm balance sheet and income statement for the entire private sector except firms in the agricultural sector

I complement my analysis with information about the structure of firms ("Contours des entreprises profilées") provided by the CASD and with national account information on depreciation rates and the price index provided by INSEE.

Sample selection From the FARE file on firms, I exclude firms with invalid information (e.g. missing ID), firms belonging to the public sector and household employers. I also drop firms from the financial sector because it is particularly challenging to estimate productivity for these firms as their income is mostly reported in their financial statement, unlike other firms. One challenge with this data is that it is reported at the legal unit level ("UL"), and several legal units can belong to the same firm. Since I want to measure job-to-job transitions across firms competing for the same workers, it is important that I aggregate firms within coherent economic units. To do so, I use information from the "Entreprise profilée" ("EP") files for available years, and extrapolate the information back in time when necessary.

From the DADS file, I exclude interns and apprenticeships as well as workers from the public sectors or working for non-profits. I keep prime-age workers (25 to 55 years old) and workers with full-time positions and permanent contracts (CDI). I focus on relatively stable jobs because I study the problem of worker retention, and it would not fit very well the case of temporary contracts (CDD) since they usually end after a short period of time. In my sample I find that full-time workers with permanent contracts account for about 60% of private sector jobs.

Definition of sectors I use detailed information about a firm industry to define a sector. Specifically, I use the NAF Rev 2. APE 2 classification, which contains about 81 sectors, including 24 sectors within manufacturing. Examples of sectors are pharmaceutical industry, retail trade or restaurants. My dataset features 6,761 workers and 1,751 per sector on average.

One advantage of using industry classifications instead of occupation or location to define a labor market is that industry is reported at the firm level, and it is therefore easy to aggregate and compute productivity at the sector level. By contrast, several employees within the same firm might belong to different occupations or live in different locations. The main drawback of using sectors is that workers might change sector when they change jobs more than they change occupation or location. I am working on a model extension with imperfect labor mobility across sectors to assess whether my results are robust to this modification.

The industry classification is available at different levels of aggregation. I choose an intermediate definition for two reasons. First, with more aggregated definition (e.g. manufacturing), changes in the sector productivity are likely affecting the entire economy. It is then more difficult to justify in my model that the interest rate is independent of sector productivity. With a more granular classification, idiosyncratic shocks to sector productivity are more likely to cancel out in the aggregate. At the other extreme, with the most granular definition most sectors are made of a few firms only and estimates of sector productivity (the average across firms) become contaminated by firm-level changes in productivity. In this case estimates of pass-through against sectoral productivity shocks are biased because they reflect the response of wages to firm-level productivity shocks. I run simulations to ensure that with the classification that I use (APE 2) and given the volatility of firm and sectoral productivity and the size of firms in my sample, this small-sample bias is negligible.

Definition of labor productivity I measure labor productivity as value added per worker, adjusted for the cost of capital

$$\text{LP} = \frac{\text{sales + variation in shocks - cost of materials - cost of capital}}{\text{number of employees}}$$

Sales includes products, services and merchandises sold while the number of employees is the average full-time equivalent number of workers in that year. The data contains information about depreciation costs reported by firms, but this information is known to be sensitive to accounting strategies followed by firms. Instead, I construct my own estimates for the cost of capital as follows. I first measure the depreciation rate at the year-industry level using national accounts data on consumption and stock of fixed capital (average of 6.5% annual). I then add the average interest rate paid by firms on their debt in my dataset for firms with positive debt (average of 10%) and multiply with firm tangible assets reported in the firm data.

I residualize the log productivity on dummies for firm-age to control for a life-cycle component. My measure of labor productivity is closely related to the accounting measure of operating profits, and therefore not surprisingly their correlation is very strong both across firms and over time within firms.

I decompose labor productivity into an aggregate, a sectoral and a firm component by assuming that they are log-additive

$$\log y_{jst} = \log a_t + \log z_{st} + \log x_{jst} \tag{A.1}$$

I measure aggregate productivity $\log a_t$ by average across firms each year. I then measure sectoral productivity $\log z_{st}$ by averaging the residual across firms within sector each year. Finally, firm-level productivity $\log x_{jst}$ is estimated as the residual. In ongoing work I investigate how my results change with alternative assumptions about productivity; for example one in which sectoral productivity is a function of aggregate productivity but with different loading coefficients. I confirm visually that there are no trends in sectoral productivity.

Definition of wages I define wages as daily labor earnings using the worker total worker earnings net of payroll taxes but gross of income taxes. This includes regular wages, overtime pay, bonuses and even payment in kind. It excludes however stock options, but these are less omnipresent in France than they are in the U.S. Note also that medical insurance is not a major component of pay in France, unlike in the U.S.

I divide total labor earnings in a year by the number of days worked at that firm. The data contains information about hours but for workers with full-time jobs and permanent contracts it usually refers to the legal number of hours and therefore does not represent the actual number of hours worked. For this reason I do not adjust for it.

Definition of labor market flows Identifying job-to-job transitions is challenging because workers sometimes hold multiple jobs at the same time. For this reason, I first identify the main job of a worker defined as the job with the earliest start date. I drop jobs that lasted for less than 35 hours during a year (a regular work week) and main jobs if they end up accounting for less than 50% of total earnings from simultaneous jobs. I also drop individuals with more than 5 jobs in a given year.

I use the exact start and end dates of jobs to identify a job transition. A job-to-job transition occurs if the new job starts 18 days or less after the previous job ends. This leaves a little bit of room for workers who take 2 weeks of holidays in between jobs. The risk is that it might also include workers who transit through unemployment for just 2 weeks and find a new job quickly. Note however that France is a country in which the job finding rate is fairly low (I estimate 20% per quarter) so most likely this risk is minimal. I also count as job-to-job transitions if the new and old jobs overlap for some time (i.e. the worker holds 2 jobs for some time), but my results are robust to remove them from the sample.

An important moment that I target in my quantitative exercise is the share of job-to-job transitions with positive wage growth. This moment is important because it is informative about why workers change jobs, and therefore has important implications for the retention elasticity. In France it is common for workers to change jobs to receive severance payments and compensations for vacations not taken when they switch job. As a result, average daily earnings at the current job is often larger than average daily earnings at the next job because it includes these extraordinary payments on top of the wage. Indeed, I compute that only 40% of workers experience a positive

	Number per year	Average duration in sample in years
Workers	532,005	7.89
Firms	129,576	8.16

Table A.1: Sample description

Avg. age	Shale male	Avg. firm size (firm obs)	Avg. firm size (worker obs)
38.4	66%	66.7	8999

Table A.2: Sample characteristics

wage growth when daily earnings are computed in this naive way, and I find that workers who are about to make a job-to-job transition experience an average wage growth of 8%, compared to 1% for the entire population. To control for these exceptional payments, I compute the share of job transitions with a positive wage change by comparing daily labor earnings at the new job with daily labor earnings at the previous job the previous year. I use the same method in the model.

When a worker separates from their previous jobs and does not make a job-to-job transition, I define it as a separation into non-employment. When a worker from my sample moves to another job that is not in my sample (e.g. transition from private sector to public sector), I do not count it either as a job-to-job transition nor as a separation into non-employment nor as a stayer.

I compute the duration of non-employment as the number of months until a worker reappears in my sample, conditional on the worker reappearing. By conditioning on whether a worker ever comes back in my sample I sort out workers who leave the labor force permanently (e.g. retirement, death). I only estimate this moment on the first half of my sample (2008-2015) so that workers have plenty of time to come back.

Summary statistics I merge the worker and firm data together and find that 95% of workers are successfully matched to a firm. I restrict my sample to workers and firms who at in the panel for at least 3 years, for firms with at least 3 employees (in the panel or not) and I keep sectors with at least 20 employees and 3 firms per year. I drop firms with negative or missing labor productivity and those with labor productivity growth below and above the 0.5 and 99.5 percentiles respectively. I also drop individuals with wage growth below or above the 0.5 and 99.5 percentiles.

A.2 Estimation of standard errors by Block bootstrap

The estimation of the moments used in the quantitative analysis is done in several steps, and for this reason I estimate standard errors by bootstrap. I sample firms with replacement and keep all the years and workers associated with a firm if it is sampled. I then create 1,000 samples and then apply my estimation procedure in each of them, including rezidualizing productivity, removing outliers, computing sectoral productivity or estimating the pass-through. The estimates that I report and their standard errors are the average estimates across bootstrap samples and the standard deviation across sample.

	Booms	Downturns
Pass-through of firm-level shocks	4.63% (1.00%)	4.88% (0.75%)
Pass-through of sectoral shocks	16.81% (4.70%)	20.1% (5.41%)

Table A.3: Pass-through estimates in boom vs. downturns

	p1	p5	p10	p25	p50	p75	p90	p95	p99	mean	s.d.
Real wage growth	- 0.57	- 0.25	- 0.13	- 0.038	0.011	0.071	0.18	0.28	0.58	0.015	0.18

Table A.4: Distribution of real annual wage growth

A.3 Additional moments

Pass-through estimates in boom vs. downturns Table A.3 reports the pass-through of firm-level and sectoral productivity shocks to wages estimated separately in booms, and in downturns. I define booms as periods in which sector-productivity in level is higher than the mean, and downturns as the complement. The estimates for the pass-through of firm shocks are meant to capture how idiosyncratic risk faced by workers vary over the cycle. The results show that the pass-through in booms and in downturns are not significantly different from one another, which is consistent with my model calibrated for France.

Dispersion in earnings growth Table A.4 describes the distribution of real annual wage growth in the data for workers continuously employed at the same firm between year $t-1$ and year t. The mean annual wage growth is 1.5% and the distribution is remarkably symmetric. The standard deviation is 0.18, the vast majority of which cannot be accounted for by observable worker characterized. Specifically, I estimate

$$\Delta \log w_{ijst} = \alpha + X'\beta + \epsilon_{ijst}$$

where X is a vector of worker characteristics, including a polynomial in experience (age minus 20), dummies for gender and firm as well as dummies for occupation (4-digit), industry (4-digit) and commuting zones. The R^2 from this regression is only 0.011.

A.4 Aggregate shocks

In this paper I focus on the behavior of sectoral $\log z_{st}$ and firm productivity $\log x_{jst}$ because I want to isolate the effects of the cyclical competition from workers from that of time-varying price of risk. The assumption underlying this approach is that sectoral productivity shocks are diversifiable for firm owners because sectors are sufficiently small. Changes in aggregate productivity $\log a_t$ on the other hand cannot be diversified and will therefore influence both the cyclicality of the competition for workers and the ability of firms to provide insurance against these shocks. This distinction is especially important in the context of wage contracts because we know that there is perfect risk-sharing if workers and firms are both equally risk-averse and the contract is not subject to any friction.

	Response of wages (OLS)	Response of job-to-job transitions (OLS)	Variance of shocks
Firm shocks	0.011 (0.002)	- 0.0031 (0.0025)	0.0887 (0.0007)
Sectoral shocks	0.043 (0.015)	0.034 (0.028)	0.0032 (0.0004)
Aggregate shocks	0.138 (0.017)	0.321 (0.034)	0.00042 (0.00002)

Table A.5: The response of wages and job-to-job transitions to firm, sectoral and aggregate shocks

Sectoral shocks also have the advantage that there is more data we can use to estimate these moments. For example in my data I have one time series of 12 years for aggregate shocks, but a panel of 81 sectors for sectoral shocks. I am working on extending my sample to the 1990s in order to get better estimates for aggregate and sectoral shocks. Doing so is difficult because there is an important break in the firm dataset in 2008 due to changes in the survey methodology.

Table A.5 shows the response of wages and job-to-job transitions to firm, sectoral and aggregate productivity shocks estimated using equation A.1 as well as estimates of the variance of these different shocks. Aggregate shocks have an even higher pass-through than sectoral shocks, and an even more cyclical response of job-to-job transitions to shocks. This supports the idea that studying sectoral shocks is informative about the nature of aggregate cycles, since the coefficients move in the same direction relative to firm-level shocks, but that they are also different, since the response is much larger to aggregate shocks. The difference in the cyclicality of job-to-job transitions with respect to sectoral and aggregate shocks might also be informative for other line of research, such as the literature on unemployment volatility (Shimer, 2005).

Table A.5 also shows that the variance of sectoral shocks is about 10 times larger than the variance of aggregate shocks, which suggest that sectoral shocks might be a larger source of risk than aggregate shocks.

A.5 Estimates of firing costs

I update estimates on firing costs from Bentolila and Bertola (1990) using data from the International Labor Organization to discipline the degree of commitment of firms Φ. In appendix C, they define firing costs as

$$\Phi = N + (1-p_a)SP + p_a\left[(1-p_u)(SP+LC) + p_u(UP+LC)\right]$$

where N represents pay during the notice period, SP is the severance payment, LC are legal costs and UP are dismissal costs if the layoff is deemed unjustified in court. p_a is the probability that the layoff is brought to court, and p_u the probability that courts rule in favor of workers. This firing costs is evidently difficult to estimate since we do not have precise information about all of these elements, especially the probabilities p_a and p_u or legal fees.

Table A.6 reports information about layoff costs from the International Labor Organization, as well as the estimates that I use in this paper. I report estimates for a worker with an average of 8 years of tenure, which is the average tenure in my sample $(1/(\text{J2J} + \text{EU}))$.

I find that firing costs account for 4.2 months in France and 0.41 months in the U.S. on average,

[8] The maximum is \$50K to \$300K depending on the firm size. For the intermediate value of \$100K, we get $UP = 25$ given a median monthly income of about \$4K.

	France	United States
Notice period N (in months of pay)	2	0
Severance pay SP (in months of pay)	2	0
Redress costs UP (in months of pay)	10	25[8]
Probability of going to court p_a (from Bentolila and Bertola (1990))	0.05	0.05
Probability worker wins p_u (from Bentolila and Bertola (1990))	0.25	0.25
Legal costs LC (from Bentolila and Bertola (1990))	2	2
Estimates of firing costs (in months of pay)	4.2	0.41
Firing costs Φ (in model units)	1.75	0.175

Table A.6: Estimates of firing costs in France and the U.S.

or about 2 weeks. The average wage in the model is 1.27 per quarter so my estimates for firing costs are $\Phi = 4.2 \times 1.27/3 \approx 1.75$ and $\Phi \approx 0.175$. Bentolila and Bertola (1990) report much higher estimates for France, of 8.2 months for the 1960s and 11 months for the 1970-80s because they use much higher severance payments for these periods.

The stringency of layoff restrictions in France relative to the U.S. is consistent with indicators published by the Organization of Economic Co-operation and Development. The measure of firing costs published by the OECD is more comprehensive than the one I use, but it is more difficult to translate it into model parameters because it is an index.

B Model appendix

B.1 Quantitative model

In this appendix, I write the laws of motions of distributions. Define the retention probability as

$$p(V_t, s_t) = \left(1 - \mathbb{E}_{\xi_t}\left[\kappa\lambda_w(v(V_t, s_t, \xi_t), z_t)\right]\right)(1-\delta)(1 - q(V_t, s_t))$$

The distribution of employed workers satisfies

$$\begin{aligned}
\psi_t(V_t, \overline{x}, x_t) = & \int_{V_{t-1}} \int_{x_{t-1}} \psi_{t-1}(V_{t-1}, \overline{x}, x_{t-1}) p(V_{t-1}, s_{t-1}) \pi_x(x_t|x_{t-1}) \mathbb{1}\left\{V_t = V(V_{t-1}, s_{t-1}, s_t)\right\} dx_{t-1} dV_{t-1} \\
& + \psi^u_{t-1} \lambda(v_u(z_{t-1}), z_{t-1}) \pi_{x_1}(x_t) \pi_{\overline{x}}(\overline{x}) \mathbb{1}\left\{V_t = V_1(v_u(z_{t-1}), z_{t-1}, s_t)\right\} \\
& + \int_{V_{t-1}} \int_{x_{t-1}} \int_{\overline{x}} \int_{\xi_{t-1}} \psi_{t-1}(V_{t-1}, \overline{x}, x_{t-1}) \kappa\lambda(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1}) \times \\
& \qquad \times \pi_\xi(\xi_{t-1}) \pi_{\overline{x}}(\overline{x}) \pi_{x_1}(x_t) \mathbb{1}\left\{V_t = V_1(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1}, s_t)\right\} d\xi_{t-1} d\overline{x} dx_{t-1} dV_{t-1}
\end{aligned}$$

and the distribution of unemployed workers satisfies

$$\begin{aligned}
\psi^u_t = \psi^u_{t-1}\left(1 - \lambda(v_u(z_{t-1}), z_{t-1})\right) + & \int_{V_{t-1}} \int_{x_{t-1}} \int_{\overline{x}} \left(1 - \mathbb{E}_{\xi_{t-1}}\left[\kappa\lambda_w(v(V_{t-1}, s_{t-1}, \xi_{t-1}), z_{t-1})\right]\right) \times \\
& \times \left(\delta + (1-\delta) q(V_{t-1}, s_{t-1})\right) \psi_{t-1}(V_{t-1}, \overline{x}, x_t) d\overline{x} dx_{t-1} dV_{t-1}
\end{aligned}$$

where $\pi_x(x_t|x_{t-1})$ is the probability of x_1 given x_{t-1}, $\pi_{x_1}(x_t)$ is the probability that firm productivity is x_t during the first period of production and $\pi_{\overline{x}}(\overline{x})$ is the probability that fixed productivity is $\overline{x}$. The first term contributing to $\psi_t(V_t, \overline{x}, x_t)$ represents stayers, the second represents new hires from unemployment and the third new hires from employment. The first term contributing to ψ^u_t represents unemployed workers who did not find a job, and the second term represents quits and exogenous separations.

B.2 Continuous time model

B.2.1 Environment

I describe the environment of the model studied in section 4.

Workers have utility $u(w)$ and have no accesss to financial markets. Their discount rate is r. Firms are owned by risk-neutral investors with discount rate r. Output $\exp(x+z)$ is produced within matches with firm productivity x and sectoral productivity z following

$$dx_t = -\alpha_x x_t dt + \sigma_x dB_{xt} \quad \text{and} \quad dz_t = -\alpha_z z_t dt + \sigma_z dB_{zt}$$

where B_{xt} and B_{zt} are Brownian motions.

Search is directed so wokers apply for jobs in labor markets indexed by the present value that a worker would get v. The job finding probability follows a Poisson process with intensity $\kappa\lambda_w(v, z)$,

$$P(\tau > t) = \exp\left(-\kappa \int_0^t \lambda_w(v_s, z_s) ds\right)$$

where τ denotes the stopping time describing when the worker finds another job.

Contracts Contracts specify a wage for each history of firm and sectoral productivity shocks

$$w_t(\{x_s, z_s; 0 \le s \le t\})$$

After signing the contract, the worker chooses a job search strategy

$$v_t(\{x_s, z_s; 0 \leq s \leq t\})$$

to maximize expected utility.

Worker value function Given contract w and search strategy v, the value of a worker is

$$V^v(w) = \mathbb{E}\left[\int_0^\tau e^{-rt}u(w_t)dt + e^{-r\tau}v_\tau\right]$$

where the expectation is taken over the paths of (B_{xt}, B_{zt}, τ^v). We can rewrite this as

$$\begin{aligned}
V_0^v(w) &= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(\mathbb{1}(\tau^v > t)u(w_t) + \mathbb{1}(\tau^v = t)v_t\right)dt\right] \\
&= \mathbb{E}\left[\int_0^\infty e^{-rt}\left(P(\tau^v > t)u(w_t) + P(\tau^v = t)v_t\right)dt\right] \\
&= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right)dt\right]
\end{aligned}$$

where the second line uses the fact that B_{xt}, B_{zt}, τ^v are mutually independent and the last line uses the definition of τ. The expectation in the last line is taken over the paths of (B_{xt}, B_{zt}). In general, the value of a worker at time t is

$$V_t^v(w) = \mathbb{E}_t\left[\int_t^\infty \exp\left(-r(h-t) - \kappa\int_t^h \lambda_w(v_s, z_s)ds\right)\left(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h\right)dh\right]$$

Therefore, the value of a contract for a worker is defined as

$$V = \max_v V_0^v(w)$$

Optimal contract Given contract w and search strategy v, the value of a firm is

$$\begin{aligned}
\Pi^v(w) &= \mathbb{E}\left[\int_0^\tau e^{-rt}\left(\exp(x_t + z_t) - w_t\right)dt\right] \\
&= \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(\exp(x_t + z_t) - w_t\right)dt\right]
\end{aligned}$$

Given the job finding rate $\lambda(v, z)$ and the processes for productivity x and z, the optimal contract solves

$$\begin{aligned}
\Pi(V_0, x_0, z_0) = \max_w \quad & \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(\exp(x_t + z_t) - w_t\right)dt\right] \\
\text{s.t.} \quad & V_0 = \mathbb{E}\left[\int_0^\infty \exp\left(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds\right)\left(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t\right)dt\right] \\
& v \in \arg\max_v V^v(w)
\end{aligned}$$

B.2.2 Recursive formulation of the contract

In this section I use methods introduced in Sannikov (2008) to write this contract recursively. For simplicity of notations I write $V_t^v(w) = V_t^v$. I first derive the law of motion of the worker promised value V_t^v for any contract and strategy.

Lemma 2. *Given a contract w and a search strategy v, the worker value V_t^v satisfies*

$$dV_t^v = (rV_t^v - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v))\,dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt}$$

for some stochastic processes Δ_{xt}, Δ_{zt}.

Proof. Define the process H_t^v as

$$H_t^v = \int_0^t R_h^v\,(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h)\,dh + R_t^v V_t^v \tag{A.2}$$

where $R_t^v \equiv \exp(-rt - \kappa\int_0^t \lambda_w(v_s, z_s)ds)$ is the effective discount rate. Notice that

$$\begin{aligned} \mathbb{E}\left[H_t^v\right] = &\ \mathbb{E}\left[\int_0^t R_h^v\,(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h)\,dh\right] + \mathbb{E}\left[\mathbb{E}_t\left[\int_t^\infty R_h^v\,(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h)\,dh\right]\right] \\ = &\ \mathbb{E}\left[\int_0^\infty R_h^v\,(u(w_h) + \kappa\lambda_w(v_h, z_h)v_h)\,dh\right] = H_0^v \end{aligned}$$

so H_t^v is a Martingale with respect to the filtration generated by x and z. By the Martingale representation theorem, there exist processes Δ_{xt}, Δ_{zt} such that

$$dH_t^v = \Delta_{xt}R_t^v\sigma_x dB_{xt} + \Delta_{zt}R_t^v\sigma_z dB_{zt}$$

Now using Ito's lemma on equation A.2 we find

$$dH_t^v = R_t^v\,(u(w_t) + \kappa\lambda_w(v_t, z_t)v_t)\,dt - R_t^v\,(r + \kappa\lambda(v_t, z_t))\,V_t^v dt + R_t^v dV_t^v$$

Combining the two expressions for dH_t^v gives

$$dV_t^v = (rV_t^v - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v))\,dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt}$$

This concludes the proof. □

The next lemma characterizes incentive compatible strategies v in terms of the worker continuation value V_t^v.

Lemma 3. *A strategy v is incentive compatible if*

$$v_t \in \arg\max_v\ \lambda_w(v, z_t)\,(v - V_t^v)$$

Proof. Let v be an incentive compatible search strategy. We show that deviations are not profitable at any t. Assume that the worker deviates to an alternative strategy $\hat{v}$ until time t. Define $H_t^{\hat{v}}$ the process corresponding to this deviation,

$$H_t^{\hat{v}} = \int_0^t R_h^{\hat{v}}\,(u(w_h) + \kappa\lambda_w(\hat{v}_h, z_h)\hat{v}_h)\,dh + R_t^{\hat{v}} V_t^v \tag{A.3}$$

where the continuation value at time t is V_t^v because the worker follows the recommended strategy thereafter.

Note that $H_0^{\hat{v}} = H_0^v$. We want the process H_t^v to be a martingale under v and a super-martingale under any alternative strategy $\hat{v}$ so that $\mathbb{E}[H_t^v] = H_0^v = H_0^{\hat{v}} \geq \mathbb{E}[H_t^{\hat{v}}]$. This ensures that the worker will never choose to deviate from search strategy v since this would lower her expected utility $\mathbb{E}[H_t^{\hat{v}}]$. Using the law of motion for V from lemma 2 and equation A.3 we get

$$\begin{aligned} dH_t^{\hat{v}} &= R_t^{\hat{v}}\left(u(w_t) + \kappa\lambda_w(\hat{v}_t, z_t)\hat{v}_t\right)dt - R_t^{\hat{v}}\left(r + \kappa\lambda_w(\hat{v}_t, z_t)\right)V_t^v dt + R_t^{\hat{v}} dV_t^v \\ &= R_t^{\hat{v}}\left(\kappa\lambda_w(\hat{v}_t, z_t)(\hat{v}_t - V_t^v) - \kappa\lambda_w(v_t, z_t)(v_t - V_t^v)\right)dt + R_t^{\hat{v}}\Delta_{xt}\sigma_x dB_{xt} + R_t^{\hat{v}}\Delta_{zt}\sigma_z dB_{zt} \end{aligned}$$

$H_t^{\hat{v}}$ is a super-martingale if and only its drift is negative, i.e.

$$\lambda_w(v_t, z_t)\,(v_t - V_t) \geq \lambda_w(\hat{v}_t, z_t)\,(\hat{v}_t - V_t) \quad \text{for all } \hat{v}_t$$

This can be written as

$$v_t \in \arg\max_v \ \lambda_w(v, z_t)\,(v - V_t^v)$$

This concludes the proof. □

Using lemmas 2 and 3 we can rewrite the optimal contracting problem as

$$\Pi(V_t, x_t, z_t) = \max_{w, \Delta_x, \Delta_z} \quad \mathbb{E}\left[\int_0^\infty e^{-rt - \int_0^t \kappa\lambda_w(v_s, z_s)ds}(\exp(x_t + z_t) - w_t)dt\right]$$

subject to

$$\begin{aligned} &\text{(PK)}: \quad dV_t = \left(rV_t - u(w_t) - \kappa\lambda_w(v_t, z_t)(v_t - V_t)\right)dt + \Delta_{xt}\sigma_x dB_{xt} + \Delta_{zt}\sigma_z dB_{zt} \\ &\text{(IC-v)}: \quad v_t \in \arg\max_v \ \lambda_w(v, z_t)\,(v - V_t) \end{aligned}$$

B.2.3 Proofs for section 4

It will be useful to write the HJB corresponding to the optimal contract

$$\begin{aligned} (r + \kappa\lambda_w(v(V,z), z))\Pi(V,x,z) \quad &= \max_{w, \Delta_x, \Delta_z} \exp(x+z) - w \\ &+ \left(rV - u(w) - \kappa\lambda_w(v(V,z),z)(v(V,z) - V)\right)\Pi_V(V,x,z) \\ &- \alpha_x x \Pi_x(V,x,z) - \alpha_z z \Pi_z(V,x,z) \\ &+ \sigma_x^2\left[\tfrac{1}{2}\Delta_x^2\Pi_{VV}(V,x,z) + \tfrac{1}{2}\Pi_{xx}(V,x,z) + \Delta_x\Pi_{Vx}(V,x,z)\right] \\ &+ \sigma_z^2\left[\tfrac{1}{2}\Delta_z^2\Pi_{VV}(V,x,z) + \tfrac{1}{2}\Pi_{zz}(V,x,z) + \Delta_z\Pi_{Vz}(V,x,z)\right] \end{aligned}$$

Proposition 1 I first derive the optimal path of wages since it does not require my approximation in the degree of search efficiency $\kappa \to 0$.

Proof. The optimality conditions of the HJB with respect to w is

$$w(V,x,z) = (u')^{-1}\left(-\frac{1}{\Pi_V(V,x,z)}\right) \tag{A.4}$$

and with respect to , Δ_x and , Δ_z are

$$\Delta_x(V,x,z) = -\frac{\Pi_{Vx}(V,x,z)}{\Pi_{VV}(V,x,z)}, \quad \Delta_z(V,x,z) = -\frac{\Pi_{Vz}(V,x,z)}{\Pi_{VV}(V,x,z)} \tag{A.5}$$

Applying Ito's lemma on the optimality condition for w gives

$$dw_t = -\frac{w_t u'(w_t)}{\gamma(w_t)} d\Pi_V(V_t, X_t) \tag{A.6}$$

and applying Ito's lemma on $F_V(V, x, z)$ gives

$$\begin{aligned} d\Pi_V = & \left(\mu_V \Pi_{VV} + \mu_x \Pi_{Vx} + \mu_z \Pi_{Vz}\right) dt \\ & + \left(\tfrac{1}{2}\left(\Delta_x^2 \sigma_x^2 + \Delta_z^2 \sigma_z^2\right) \Pi_{VVV} + \sigma_x^2 \left(\tfrac{1}{2}\Pi_{Vxx} + \Delta_x \Pi_{VVx}\right) + \sigma_z^2 \left(\tfrac{1}{2}\Pi_{Vzz} + \Delta_z \Pi_{VVz}\right)\right) dt \end{aligned} \tag{A.7}$$

where we used optimality condition for $\Delta_x(V, X)$ to get ride of the diffusion terms. The terms μ_V, μ_x and μ_z denote the drift of V, x and z.

Differentiate the HJB equation with respect to V gives

$$\begin{aligned} \kappa \frac{\partial \lambda_w(v(V,z),z)}{\partial V} \Pi & = \mu_V \Pi_{VV} + \mu_x \Pi_{Vx} + \mu_z \Pi_{Vz} \\ & + \tfrac{1}{2}\left(\Delta_x^2 \sigma_x^2 + \Delta_z^2 \sigma_z^2\right) \Pi_{VVV} + \sigma_x^2 \left(\tfrac{1}{2}\Pi_{Vxx} + \Delta_x \Pi_{VVx}\right) + \sigma_z^2 \left(\tfrac{1}{2}\Pi_{Vzz} + \Delta_z \Pi_{VVz}\right) \end{aligned}$$

where we used the envelope theorem to get $\partial_V \lambda_w(v(V, z), z)(v(V, z) - V) = -\lambda_w(v(V, z), z)$. Combining this expression with A.6 and A.7 gives

$$dw_t = -\frac{w_t u'(w_t)}{\gamma(w_t)} \Pi(V_t, x_t, z_t)\, \kappa \frac{\partial \lambda_w(v(V, z), z)}{\partial V} dt + 0 \times dB_{xt} + 0 \times dB_{zt}$$

Rewriting this expression using the retention elasticity (7) gives the desired result. □

Lemma 1 Before solving the for firm value when $\kappa \to 0$, I solve it when $\kappa = 0$.

Lemma 4. *If $\kappa = 0$, wages are constant and the firm value is*

$$\Pi(V, x, z) = g(x, z) - h(V)$$

where $g(x, z) = r^{-1}(\exp(x + z) + \mathcal{D}g(x, z))$ is the present value of output and $h(V) = u^{-1}(rV)/r$ is the cost of providing a value V to workers. The policy functions are

$$w(V) = u^{-1}(rV), \quad \Delta_x = 0, \quad \Delta_z = 0$$

Proof. We prove this result by guess and verify. Conjecture that the firm value takes the form

$$F(V, x, z) = g(x, z) - h(V)$$

for two functions $g(x, z)$ and $h(V)$ to be determined. This implies that $\Delta_x = \Delta_z = 0$ and that $w(V, x, z) = w(V)$ from equations (A.4) and (A.5). Plugging this conjecture in the HJB together with $\kappa = 0$ gives two conditions that $g(x, z)$ and $h(V)$ must satisfy

$$\begin{aligned} rg(x, z) & = \exp(x + z) + \mathcal{D}g(x, z) \\ rh(V) & = w(V) + (rV - u(w(V)))\, h'(V) \end{aligned}$$

Since these equations are independent, this confirms our guess. It is straightforward to verify that $h(V) = u^{-1}(rV)/r$ and $w(V) = u^{-1}(rV)$ satisfy the second ODE. Finally, plug in the value of $w(V)$ in dV_t together with $\Delta_x = \Delta_z = 0$ to show that the worker value and therefore the wage are constant over time. □

I now prove lemma 1.

Proof. Consider a first-order Taylor expansion of $\Pi(V,x,z)$ around $\kappa = 0$

$$\Pi(V,x,z) = g(x,z) - h(V) + \kappa \partial_\kappa \Pi(V,x,z;\kappa = 0) + O(\kappa^2) \tag{A.8}$$

using lemma 4. Introduce the function

$$\ell(V,x,z) \equiv -\kappa^{-1}\left[\Pi(V,x,z) - (g(x,z) - h(V))\right] \tag{A.9}$$

and therefore $\partial_\kappa \Pi(V,x,z;\kappa = 0) = \lim_{\kappa \to 0} \ell(V,x,z)$. The function $\ell(V,x,z)$ can be interpreted as the cost of retaining workers to the firm, scaled by the degree of worker mobility κ.

Plugging (A.9) in the HJB and using the optimality conditions (A.4) and (A.5) gives

$$\begin{aligned}
0 \ &= \exp(x+z) - w(V,x,z) \\
&+ (rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z) - V))\,(-h'(V) - \kappa\ell_V(V,x,z)) \\
&-\alpha_x x g_x(x,z) - \alpha_z z g_z(x,z) \\
&+\frac{\sigma_x^2}{2}\frac{(g_{xx}(x,z) - \kappa\ell_{xx}(V,x,z))(h''(V) - \kappa\ell_{VV}(V,x,z)) + \kappa^2\ell_{Vx}^2(V,x,z)}{h''(V) + \kappa\ell_{VV}(V,x,z)} \\
&+\frac{\sigma_z^2}{2}\frac{(g_{zz}(x,z) - \kappa\ell_{zz}(V,x,z))(h''(V) - \kappa\ell_{VV}(V,x,z)) + \kappa^2\ell_{Vz}^2(V,x,z)}{h''(V) - \kappa\ell_{VV}(V,x,z)} \\
&-(r + \kappa\lambda_w(v(V,z),z))\,(g(x,z) - h(V) - \kappa\ell(V,x,z))
\end{aligned}$$

We now subtract $rg(X)$ and $rh(V)$ on both sides and use their definition from lemma 4 to get

$$\begin{aligned}
-(r + \kappa\lambda_w(v(V,z),z))\kappa\ell(V,x,z) \ &= \kappa\lambda_w(v(V,z),z)h(V) - \kappa\lambda_w(v(V,z),z)g(x,z) - [w(V,x,z) - w(V)] \\
&- (rV - u(w(V,x,z)) - \kappa\lambda_w(v(V,z),z)(v(V,z) - V))\,\kappa\ell_V(V,x,z) \\
&+\kappa\lambda_w(v(V,z),z)[v(V,z) - V]h'(V) + [u(w(V,x,z)) - u(w(V))]\,h'(V) \\
&-\kappa\alpha_x x\ell_x(V,x,z) - \kappa\alpha_z z\ell_z(V,x,z) \\
&-\frac{\sigma_x^2}{2}\frac{\kappa\ell_{xx}(V,x,z)(h''(V) - \kappa\ell_{VV}(V,x,z)) - \kappa^2\ell_{Vx}^2(V,x,z)}{h''(V) - \kappa\ell_{VV}(V,x,z)} \\
&-\frac{\sigma_z^2}{2}\frac{\kappa\ell_z(V,x,z)(h''(V) - \kappa\ell_{VV}(V,x,z)) - \kappa^2\ell_{Vz}^2(V,x,z)}{h''(V) - \kappa\ell_{VV}(V,x,z)}
\end{aligned} \tag{A.10}$$

where $w(V)$ is the wage policy when $\kappa = 0$.

Consider the term

$$\lim_{\kappa \to 0} \frac{w(V,x,z) - w(V)}{\kappa} = \lim_{\kappa \to 0} \frac{1}{\kappa}\left((u')^{-1}\left(\frac{1}{h'(V) - \kappa\ell_V(V,x,z)}\right) - (u')^{-1}\left(\frac{1}{h'(V)}\right)\right)$$

Take a Taylor expansion of the first term around $\kappa = 0$

$$(u')^{-1}\left(\frac{1}{h'(V) - \kappa\ell_V(V,x,z)}\right) = (u')^{-1}\left(\frac{1}{h'(V)}\right) + \kappa\frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))} + O(\kappa^2)$$

where we used $h'(V) = 1/u'(w(V))$. Therefore,

$$\lim_{\kappa \to 0} \frac{w(V,x,z) - w(V)}{\kappa} = \frac{\ell_V(V,x,z)}{h'(V)}\frac{u'(w(V))}{u''(w(V))}$$

Similarly, we get

$$\lim_{\kappa \to 0} \frac{u(w(V,x,z)) - u(w(V))}{\kappa} = -\frac{\ell_V(V,x,z)}{h'(V)^2}\frac{u'(w(V))}{u''(w(V))}$$

Now divide equation A.10 by κ and take limit as $\kappa \to 0$

$$\begin{aligned} r\ell(V,x,z) &= -\lambda_w(v(V,z),z)h(V) + \lambda_w(v(V,z),z)g(x,z) \\ &\quad -\lambda_w(v(V,z),z)[v(V,z)-V]h'(V) \\ &\quad -\alpha_x x \ell_x(V,x,z) - \alpha_z z \ell_z(V,x,z) \\ &\quad +\frac{\sigma_x^2}{2}\ell_{xx}(V,x,z) + \frac{\sigma_z^2}{2}\ell_{zz}(V,x,z) \end{aligned}$$

where we used $rV - u(w(V)) = 0$ from lemma 4. We can reformulate this equation using the differential operator introduced at the start of section 4 as

$$r\ell(V,x,z) = \lambda_w(v(V,z),z)\left[g(x,z) - h(V) - [v(V,z)-V]h'(V)\right] + \mathcal{D}\ell(V,x,z)$$

Rewriting this equation using the definition of $h(V)$ concludes the proof. □

Proposition 2

Proof. To first order in κ, the optimality condition with respect to Δ_x (A.5) in the HJB becomes

$$\Delta_x(V,x,z) = -\kappa \frac{\ell_{Vx}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vx}(V,x,z)$ from lemma 1. We have

$$(r+\alpha_x)\,\ell_x(V,x,z) = \lambda_w(v(V,z),z)g_x(x,z) + \mathcal{D}\ell_x(V,x,z)$$

Now differentiate with respect to V to find

$$(r+\alpha_x)\,\ell_{Vx}(V,x,z) = \frac{\partial\lambda_w(v(V,z),z)}{\partial V}g_x(x,z) + \mathcal{D}\ell_{Vx}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial V}\frac{g_x(x,z)}{h''(V)} + \mathcal{D}\Delta_x(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (7) when $\kappa \to 0$ to get

$$(r+\alpha_x)\,\Delta_x(V,x,z) = g_x(x,z)\frac{\epsilon(V,x,z)}{\gamma(w(V))}u'(w(V)) + \mathcal{D}\Delta_x(V,x,z)$$

This concludes the proof. □

Proposition 3

Proof. To first order in κ, the optimality condition with respect to Δ_z (A.5) in the HJB becomes

$$\Delta_z(V,x,z) = -\kappa \frac{\ell_{Vz}(V,x,z)}{h''(V)}$$

Now consider $\ell_{Vz}(V,x,z)$ from lemma 1. We have

$$(r+\alpha_z)\,\ell_z(V,x,z) = \lambda_w(v(V,z),z)g_z(x,z) + \tfrac{\partial\lambda_w(v(V,z),z)}{\partial z}\,(g(x,z)-h(V)) \\ -\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)\,h'(V) + \mathcal{D}\ell_z(V,x,z)$$

where I used the envelope condition of the worker search problem (6) for the third term. $\frac{\partial\lambda_w(v(V,z),z)}{\partial z}$ denotes the total derivative with respect to z whereas $\lambda_{wz}(v(V,z),z)$ denotes the partial derivative with respect to the second argument.

Differentiate with respect to V to find

$$(r+\alpha_z)\,\ell_{Vz}(V,x,z) = \tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}g_z(x,z) + \tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\,(g(x,z)-h(V)) \\ -\lambda_{wz}(v(V,z),z)\,(v(V,z)-V)\,h''(V) + \mathcal{D}\ell_{Vz}(V,x,z)$$

Multiply by $-\kappa/h''(V)$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = -\kappa\tfrac{\partial\lambda_w(v(V,z),z)}{\partial V}\tfrac{g_z(x,z)}{h''(V)} - \kappa\tfrac{\partial^2\lambda_w(v(V,z),z)}{\partial V\partial z}\tfrac{(g(x,z)-h(V))}{h''(V)} \\ +\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V) + \mathcal{D}\Delta_z(V,x,z)$$

Now use $h''(V) = r\gamma(w(V))/\left(w(V)u'(w(V))^2\right)$ and the definition of $\epsilon(V,x,z)$ from equation (7) when $\kappa\to 0$ to get

$$(r+\alpha_z)\,\Delta_z(V,x,z) = \left[g_z(x,z)\tfrac{\epsilon(V,x,z)}{\gamma(w(V))} + (g(x,z)-h(V))\,\tfrac{\epsilon_z(V,x,z)}{\gamma(w(V))}\right]u'(w(V)) \\ +\kappa\lambda_{wz}(v(V,z),z)\,(v(V,z)-V) + \mathcal{D}\Delta_z(V,x,z)$$

where $\epsilon_z(V,x,z)\equiv\partial\epsilon(V,x,z)/\partial z$. Finally, use $(g(x,z)-h(V))\,\epsilon_z(V,x,z) = \Pi(V,x,z)\epsilon_z(V,x,z)$ to first order in κ to rewrite this expression as in the proposition. □

The pass-through of firm-level shocks to wages in section 4.3 Start from the path of wages in proposition 1 and take a first-order approximation in κ. This equation becomes

$$dw_t = (rg(x_t,z_t)-w_t)\;\frac{\epsilon(V_t,w_t,z_t)}{\gamma(w_t)}dt$$

where I wrote the retention elasticity as a function of the wage directly, and to first order in κ,

$$\epsilon(V,w,z)\equiv -\kappa\frac{\partial\lambda_w(v(V,z),z)}{\partial v(V,z)}\times\frac{\partial v(V,z)}{\partial V}\times\frac{wu'(w)}{r}$$

To first order in x, we can rewrite the wage growth equation as

$$\hat{w}_t \approx \int_0^t (rg_x(x_0,z_0)\exp(-\alpha_z s)\,\hat{x}_0 - \hat{w}_s)\;\frac{\epsilon(V_0,w_0,z_0)}{\gamma(w_0)}ds$$

where we used the definition of $\hat{x}_s$ and made the additional approximation that the ratio $\epsilon(V_0,w_0,z_0)/\gamma(w_0)$ was constant over time. We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0-\alpha_x}\frac{\epsilon_0}{\gamma_0}g_x(x_0,z_0)\left[\exp(-\alpha_x t)-\exp\left(-\frac{\epsilon_0}{\gamma_0}t\right)\right]\hat{x}_0$$

where $\epsilon_0 \equiv \epsilon(V_0, w_0, z_0)$ and $\gamma_0 \equiv \gamma(w_0)$.

The pass-through of sectoral shocks to wages in section 4.4 Again start from the path of wages with $\kappa \to 0$,

$$dw_t = (rg(x_t, z_t) - w_t) \; \frac{\epsilon(V_t, w_t, z_t)}{\gamma(w_t)} dt$$

To first order in z, we can rewrite the wage growth equation as

$$\begin{aligned} \hat{w}_t \approx & \int_0^t (rg_z(x_0, z_0) \exp(-\alpha_z s) \hat{z}_0 - \hat{w}_s) \; \tfrac{\epsilon(V_0, w_0, z_0)}{\gamma(w_0)} ds \\ & + \int_0^t (rg(x_0, z_0) - w_0) \; \tfrac{\epsilon_z(V_0, w_0, z_0)}{\gamma(w_0)} \exp(-\alpha_z s) \hat{z}_0 ds \end{aligned}$$

where $\epsilon_z(V_0, w_0, z_0) \equiv \partial \epsilon(V_0, w_0, z_0)/\partial z$. This equation approximates the true wage response because we keep the ratio $\epsilon(V_0, w_0, z_0)/\gamma(w_0)$ and the firm value $rg(x_0, z_0) - w_0$ constant over time. The key difference with the pass-through from-firm level shocks derived in appendix B.2.3 is that sectoral productivity z enters directly as an input in the retention elasticity $\epsilon(V_0, w_0, z_0)$.

We can solve this ODE in $\hat{w}_t$ and find

$$\hat{w}_t \approx \frac{r}{\epsilon_0/\gamma_0 - \alpha_z} \left(g_z(x_0, z_0) \frac{\epsilon_0}{\gamma_0} + \Pi(V_0, x_0, z_0) \frac{\epsilon_{z0}}{\gamma_0} \right) \left[\exp(-\alpha_z t) - \exp\left(-\frac{\epsilon_0}{\gamma_0} t \right) \right] \hat{z}_0$$

where $\epsilon_{z0} \equiv \partial \epsilon(V_0, w_0, z_0)/\partial z$.

Special case: no aggregate shocks and mean-reverting productivity To illustrate the formulas derived in section 4, I focus on a special case in which these formulas can be derived in closed form.

Assume that sectoral productivity is constant ($\sigma_z = \rho_z = 0$), and assume that the production function is simply xz. This means that productivity follows a normal distribution, instead of a log-normal distribution as in the quantitative model. In this case, we can solve for $g(x, z)$ by guess and verify as

$$g(x, z) = \frac{zx}{r + \alpha_x}$$

so that $g_x(x, z) = z/(r + \alpha_x)$. To solve for $\Delta_x(V, x, z)$ notice that the elasticity $\epsilon(V, x, z)$ is independent of x to first order in κ. Then, by guess and verify we get

$$\Delta_x(Vx, z) = (r + \alpha_x)^{-1} g_x(x, z) \frac{\epsilon(V, x, z)}{\gamma(w(V))} u'(w(V))$$

B.3 Two-period model

I describe a 2-period version of the model that I use to characterize the contract as in section 4, and as a benchmark relative to the model with risk-free bonds in section 6.

B.3.1 Environment

In the 2-period model, I assume that workers do not have preference shocks, do not quit voluntarily into unemployment and that firms have full commitment. Matches can still separate with exogenous probability δ.

Timing At $t = 0$, a unit mass of workers is matched with firms. These workers have promised value V_0. Firm-level and sectoral productivity are normalized to 1 at $t = 0$. At the beginning of $t = 1$, firm-level and sectoral productivity shocks x and z are realized and some matches separate exogenously with probability δ. After this, firms post vacancies to poach workers from existing matches, employed workers search for new jobs and new matches are formed. At the end of each period, firms produce and workers consume their wage if employed and home production b if unemployed.

Note that the timing is slightly different than in the quantitative model because I assume that workers can switch jobs before production occurs and wages are paid.

Search and matching Workers decide in which labor market to search at the start of $t = 1$ after learning about the realization of productivity. This choice is private information to workers. Labor markets are indexed by the value that workers get in this market, denoted v. Denote the probability that a worker finds a job in market v by $\kappa\lambda_w(v,z)$ and the probability that a firm finds a job by $\lambda_f(v,z)$.

Entry of new firms at $t = 1$ is subject to a free entry condition. A new firm must pay a cost k to post a vacancy. A new firm that successfully matched with a worker in market v at $t = 1$ generates profits $x^e z - w$ where $w = u^{-1}(v)$ is the wage paid in this labor market and x^e is the firm-specific productivity of a new firm. The free entry condition states

$$-k + \lambda_f(v,z)\left(x^e z - u^{-1}(v)\right) \leq 0$$

Contract A contract specifies wages $\{w_0, w_1(x,z)\}$ at $t = 0$ and at $t = 1$ for each realization of firm-level and sectoral productivity.

Given a contract, the worker chooses the labor market in which to search to maximize utility. Specifically, at $t = 1$ the worker solves

$$\max_v \ \kappa\lambda_w(v,z)v + (1 - \kappa\lambda_w(v,z))u(w_1(x,z))$$

Denote the search policy $v(w_1(x,z),z)$. Define the retention probability $p(w,z)$ and the expected gain from job-to-job transitions $S(w,z)$ as

$$\begin{aligned} p(w,z) \equiv & \quad 1 - \kappa\lambda_w(v(w,z),z) \\ S(w,z) \equiv & \quad \kappa\lambda_w(v(w,z),z)\,(v(w,z) - u(w)) \end{aligned}$$

The optimal contract solves

$$\begin{aligned} \max_{w_0, w_1(x,z)} \quad & 1 - w_0 + (1-\delta)\beta\mathbb{E}_{x,z}\left[p(w_1(x,z),z)\,(xz - w_1(x,z))\right] \\ \text{s.t.} \quad & u(w_0) + \delta u(b) + (1-\delta)\beta\mathbb{E}_{x,z}\left[u(w_1(x,z)) + S(w_1(x,z),z)\right] \leq V_0 \end{aligned}$$

B.3.2 Characterization

Matching rate In the 2-period model it is trivial to derive the equilibrium vacancy filling rate $\lambda_f(v,z)$ because it only depends on profits today. From the free entry condition, we get

$$\lambda_f(v,z) = \frac{k}{x^e z - u^{-1}(v)}$$

This is an increasing function in v. Furthermore, the labor market with the highest value $\overline{v}$ such that firms generate positive profits by posting vacancies is $\overline{v}(z) = u(x^e z)$. As sectoral productivity increases from z_l to z_h, firms are willing to post vacancies in additional labor markets with higher value v satisfying $\overline{v}(z_l) < v \leq \overline{v}(z_h)$. As a result, workers paid relatively high wages at their current jobs are more likely to search for a new job when sectoral productivity increases.

Given the matching function, it is possible to derive the job finding rate $\lambda_w(v,z)$.

Optimal contract Combining the optimality conditions of the optimal contract gives

$$\frac{u'(w_1(x,z)) - u'(w_0)}{u'(w_0)} = -\frac{\partial_w p(w_1(x,z),z)}{p(w_1(x,z),z)} \left(xz - w_1(x,z)\right) \tag{A.11}$$

This equation is the equivalent of proposition 1 in the 2-period model. To gain more intuitions about the path of wages over time and across states, I now implement the approximation $\kappa \to 0$.

Approximate solution as $\kappa \to 0$ First solve for the optimal contract when $\kappa = 0$. The optimality condition becomes

$$\frac{u'(w_1(x,z)) - u'(w_0)}{u'(w_0)} = 0$$

so that $w_1(x,z) = w_0$, that is wages are constant over time and across states. From the promise keeping constraint, we get $w_0 = u^{-1}\left((V_0 - \delta u(b)) / (1 + (1-\delta)\beta)\right)$.

Now take a first-order approximation of equation (A.11) in κ around $\kappa = 0$. It becomes

$$w_1(x,z) - w_0 = \frac{\epsilon(w_0,z)}{\gamma(w_0)} \left(xz - w_0\right) \tag{A.12}$$

where $\gamma(w) \equiv -wu''(w)/u'(w)$ and where the retention elasticity is defined as

$$\epsilon(w,z) \equiv \frac{\partial p(w,z)}{\partial w} w = -\kappa \frac{\partial \lambda_w(v,z)}{\partial v} \times \frac{\partial v(w,z)}{\partial w} \times w$$

Equation (A.12) shows that wages grow faster when the retention elasticity $\epsilon(w,z)$, future profits $xz - w_0$ and the intertemporal elasticity of substitution $1/\gamma(w_0)$ are large.

Pass-through of firm-level productivity shock Define the pass-through of a firm-level productivity shock to wages as the difference between the wage in the high and low productivity states, $w(x^h,z) - w(x^l,z)$. From equation (A.12) we get

$$w_1(x^h,z) - w_1(x^l,z) = \frac{\epsilon(w_0,z)}{\gamma(w_0)} \left(x^h z - x^l z\right)$$

The pass-through to the value of workers is defined as $\Delta_x \equiv \left(V^h - V^l\right) / \left(x^h - x^l\right)$. This gives

$$\Delta_x(w,z) \approx z \frac{\epsilon(w_0,z)}{\gamma(w_0)} u'(w_0)$$

where we used $\kappa \to 0$ and $u(w_1(x^h,z)) - u(w_1(x^l,z)) \approx u'(w_0)\left(w_1(x^h,z) - w_1(x^l,z)\right)$. This pass-through equation for the worker value is equivalent to proposition 2 with $g_x(x,z) = z$ and where $\mathcal{D}\Delta_x = 0$ since there are no dynamics in the 2-period model.

Pass-through of sectoral productivity shock Define in an analogous way the pass-through of sectoral productivity shocks to wages as $w(x, z^h) - w(x, z^l)$. From equation (A.12) we get

$$w_1(x, z^h) - w_1(x, z^l) = \underbrace{\frac{\epsilon(w_0, z^l)}{\gamma(w_0)} \left(xz^h - xz^l\right)}_{\text{Pass-through of firm shocks}} + \underbrace{\frac{\epsilon(w_0, z^h) - \epsilon(w_0, z^l)}{\gamma(w_0)} \left(xz^h - w_0\right)}_{\text{Differential pass-through}}$$

As in the dynamic model, the differential pass-through depends on the value of firms $xz^h - w_0$ and on the cyclicality of the retention elasticity $\epsilon(w, z^h) - \epsilon(w, z^l)$.

The pass-through to the value of workers is

$$\Delta_z \approx \underbrace{\left[x\frac{\epsilon(w_0, z^l)}{\gamma(w_0)} + \frac{\epsilon_z(w_0, z^l)}{\gamma(w_0)} \left(xz^h - w_0\right)\right]}_{\text{Pass-through to wages}} u'(w_0) + \underbrace{\kappa \lambda_{wz}(v(w_0, z_l), z_l)\left(v(w_0, z_l) - u(w_0)\right)}_{\text{Change in expected gains from J2J transitions}}$$

where $\epsilon_z(w_0, z^l) \equiv \left(\epsilon(w_0, z^h) - \epsilon(w_0, z^l)\right) / (z^h - z^l)$ is the cyclicality of the retention elasticity and where $\lambda_{wz}(v(w, z), z) \equiv \frac{\partial \lambda_w(v,z)}{\partial z}|_{v=v(w,z)}$. This pass-through equation for the worker value is equivalent to proposition 3 with $g_z(x, z) = x$ and $\Pi = xz^h - w_0$.

B.4 Relation to Chetty-Baily statistic

Equation (1) in Chetty (2006) is

$$\gamma \frac{\Delta c}{c}(b^*) \approx \epsilon_{D,b}$$

where $\Delta c/c(b^*)$ is the optimal change in consumption following an unemployment shock, $\epsilon_{D,b}$ is the elasticity of the duration of unemployment D with respect to unemployment benefits b and γ is the coefficient of relative risk aversion.

Note that the duration is defined as $D = 1/(1-p)$ where p is the probability that the worker stays unemployed (the "retention probability"). Therefore,

$$\epsilon_{D,b} = \frac{dD}{db}\frac{b}{D} = \frac{dp}{db}\frac{b}{1-p} = -\epsilon_{1-p,b}$$

where $\epsilon_{1-p,b}$ is the elasticity of the job finding probability with respect to the unemployment benefit. Thus, the Chetty-Baily formula can be written as

$$\frac{\Delta c}{c} \approx -\frac{\epsilon_{1-p,b}}{\gamma}$$